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Risk Premiums versus Waiting-Options Premiums: A Simple Numerical Example

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Abstract

This paper investigates how interest rates on liquid assets and excess returns on risky assets are determined when only safe assets can be used as liquid assets when waiting for an informative signal of future payoffs. In particular, we carefully differentiate between a demand for liquid assets while waiting for new information and a demand for safe assets for precautionary reasons. Employing Kreps–Porteus preferences, numerical examples demonstrate that larger waiting-options premiums (lower interest rates) emerge with higher risk aversion in combination with more elastic intertemporal substitution.

KEYWORDS: risk premium, waiting-options premium, flexibility, Kreps–Porteus preferences, resolution of uncertainty

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1 Introduction

This paper investigates how interest rates on liquid assets and excess returns on risky assets are determined when only safe assets can be used as liquid assets when waiting for an informative signal of future payoffs. For this purpose, we carefully differentiate between a demand for liquid assets while waiting for new information[1] and a demand for safe assets for precautionary reasons.

Using a simple three-period setup, existing literature demonstrates that additional demand for liquid assets such as fiat money, public bonds, bank deposits, and inside bonds emerges when risk-hedging devices are lacking, or available only with high transaction costs. In Diamond and Dybvig (1983), for example, consumers demand privately issued deposits when insurance against idiosyncratic preference shocks is missing. Jones and Ostroy (1984) analyze a case in which investors prefer less-profitable but liquid assets to more-profitable but illiquid assets. In Holmström and Tirole (2001), private firms hold public bonds in preparation for liquidity shocks when they cannot make any short position. Dutta and Kapur (1998) present a case in which consumers hold public bonds or fiat money when profitable investment opportunities are completely irreversible. In Epstein (1980), which is adopted as the main vehicle of our framework, investors can hold only safe assets as liquid assets before new information concerning future payoffs arrives.

As Epstein (1980), Jones and Ostroy (1984) and others show, such a demand for liquid assets depends critically on the extent to which new information can resolve the uncertainty of future payoffs. More specifically, those who carry easily tradable assets can respond flexibly to the arrival of new information that helps to resolve uncertainty. The above feature of asset demand in economies with frictions never emerges when, with no transaction cost, agents can trade risky assets whose uncertainty is subject to informative signals.

In addition, Hahn (1990) emphasizes that carrying liquid assets while waiting for new information differs fundamentally from a demand for safe assets for precautionary reasons. In terms of the latter motive, for example, money

[1] In the literature, 'the resolution of uncertainty' has two related, but subtly different meanings. One meaning is that the nature itself resolves the uncertainty about underlying investment opportunities as time passes. What is implied by early (late) resolution of uncertainty here is that given an unconditional volatility, a conditional volatility diminishes quickly (slowly) as time goes by. The other meaning is that an informative signal helps to resolve uncertainty, and a decision maker exploits such a signal to make better decisions. An essential difference between the two is the former can be defined independently of preferences, but the latter cannot be. In this paper, we use 'the resolution of uncertainty' in the former sense, and 'the arrival of new information (informative signals)' in the latter. Appendix A offers the exact definition of 'informativeness.'

can be characterized as the least risky (safest) asset. From the viewpoint of the former motive, on the other hand, money is valuable because of flexibility or convenience. Hereafter, the excess return that is yielded as a consequence of risk aversion and preference for safe assets is called a *risk premium* as usual, while the excess return that is generated as a result of temporarily holding flexible assets while waiting for informative signals is a *waiting-options premium*.[2]

This paper develops a simple theoretical framework in order to distinguish systematically between risk and waiting-options premiums. For this purpose, we exploit a three-period model employed by Epstein (1980), and embed it in an overlapping generations setup. Here, the inside bonds traded between generations would serve as liquid instruments. Our investigation is unique in the sense that the generation of waiting-options premiums has often been treated less explicitly than that of risk premiums in the fields of macroeconomics and financial economics.

As mentioned before, a framework presented by Epstein (1980) assumes that agents cannot make any investment in risky assets before the arrival of new information. The assumption of complete exclusion of risky assets as an effective tool to respond to new information is more restrictive than other assumptions such as costly transactions, irreversibility, and lack of a short position. However, the most important analytical benefit of this assumption is that waiting-options premiums can be differentiated from risk premiums in a clear manner. The above-mentioned models with less restrictive assumptions concerning risky investment bear analytical costs in other dimensions.[3]

In addition, Epstein's partial equilibrium setup allows us to incorporate Kreps–Porteus preferences (Kreps and Porteus, 1978; Epstein and Zin, 1989; Weil, 1990) as in Miyazaki and Saito (2004), and analyze within a general equilibrium setup how the generation of waiting-options premiums depends on a combination of risk aversion and intertemporal substitution, which can be characterized by only Kreps–Porteus preferences. Epstein (1980) adopts a time-additive preference with constant relative risk aversion (CRRA), in which a degree of risk aversion is inversely proportional to an elasticity of intertemporal substitution, and these two preference parameters cannot be chosen independently.

[2] Hahn calls the latter type a *liquidity premium*.

[3] For example, when Jones and Ostroy (1984) and Holmström and Tirole (2001) assume risk neutrality, there is no room to analyze the effects of preferences on risk premiums. When Diamond and Dybvig (1983) and Dutta and Kapur (1998) consider only idiosyncratic shocks, there is no room to investigate the determination of risk premiums associated with aggregate shocks.

Our investigation may be located at the intersection of two different research agendas. One agenda concerns a case in which consumers with time-additive preferences behave in an economy with frictions when uncertainty of future payoffs is expected to be resolved partially. For example, Eeckhoudt et al. (2005) analyze the interaction between saving decisions and the resolution of uncertainty, when an insurance against idiosyncratic income risks is missing. Epstein and Turnbull (1980) investigate how asset pricing depends on the temporal resolution of uncertainty in the presence of informational frictions.

The other agenda concerns a situation where financial markets are complete and transaction frictions are absent, but preferences are no longer time-additive.[4] In this case, consumption-saving decisions and overall asset pricing are influenced by the time-varying volatility of investment opportunities.[5] Assuming Kreps–Porteus preferences,[6] Kandel and Stambaugh (1991) analyze how risk premiums are determined in a setup in which the endowment process follows autoregressive conditional heteroskedasticity, while Bansal and Yaron (2004) demonstrate that overall asset pricing depends on both risk aversion and intertemporal substitution when consumption volatility is stochastic.

Combining the two agendas, we explore the interaction among asset pricing, more general preferences, and the arrival of informative signals that help to resolve uncertainty. Our numerical analysis demonstrates that under Kreps–Porteus preferences, large risk aversion together with elastic intertemporal substitution generate larger demand or smaller supply of inside bonds, and yield larger waiting-options premiums.

This paper is organized as follows. Section 2 presents a model of Epstein (1980) augmented by Kreps–Porteus preferences. Section 3 embeds the model in an overlapping generations setup. Section 4 numerically explores the asset pricing implications of the model, and Section 5 discusses several theoretical and empirical implications of our numerical investigation.

[4]Backus et al. (2005) make a comprehensive survey of the application of non-time-additive preferences in the fields of macroeconomics and financial economics.

[5]As Weil (1989) and Kocherlakota (1990) demonstrate, even under Kreps–Porteus preferences, risk premiums depend only on the degree of relative risk aversion when an investment opportunity is fixed over time.

[6]In a related paper, adopting max–min preferences (another type of nonexpected preferences), Epstein and Schneider (2005) show that aversion toward ambiguity affects risk premiums.

2 A Theoretical Framework in Partial Equilibrium

Before presenting a general equilibrium model, this section briefly provides a three-period model in a partial equilibrium setup, where a model constructed by Epstein (1980) is augmented by introducing Kreps–Porteus preferences as in Miyazaki and Saito (2004).[7]

In Epstein (1980), a consumer allocates the first period endowment over three periods in the following environments: (i) he/she can trade only risk-free assets between the first and second periods, (ii) he/she can trade only risky assets between the second and third periods, and (iii) in the interim period, he/she obtains as an informative signal a random variable that is correlated with the third period risky return.

In the above setup, a consumer can expect that the uncertainty of the third period return will be resolved to some extent in the second period. Nevertheless, the ability to respond flexibly to the arrival of new information is limited severely by the inability to trade risky assets in advance.[8] In this case, only risk-free assets serve as liquid assets in transferring resources from the first period to the second. A main advantage of this setup is that we can differentiate between the precautionary saving for the third period riskiness and the saving while waiting for new information between the first and second periods. Exploiting this feature, Section 4 will analyze how the generation of premiums on risky assets differs between the two saving motives.

In addition, the current model introduces Kreps–Porteus preferences into Epstein's (1980) setup above. Accordingly, it is possible to assume a degree of relative risk aversion and an elasticity of intertemporal substitution separately. Thanks to a more general preference, we obtain a much richer characterization of saving behavior while waiting for new information. More concretely, Epstein (1980) demonstrates that such saving behavior emerges only among consumers with highly elastic intertemporal substitution (inevitably, low risk aversion

[7]See footnote 11 for the difference between Miyazaki and Saito (2004) and the current model.

[8]If a risky asset whose uncertainty is subject to informative signals can be traded with no transaction cost between the first and second periods, then the complete markets outcome emerges in this setup. The inability to trade risky assets may be a restrictive and unrealistic assumption. Instead of the inability to trade risky assets, costly transactions or irreversibility of risky assets may serve as an alternative assumption in limiting a flexible response to the arrival of new information. As discussed in the introduction, however, such assumptions may make a corresponding model extremely difficult to analyze without other simplifying assumptions such as risk neutrality and the exclusion of aggregate risks.

under time-additive preferences). In the current model, on the other hand, such waiting behavior may emerge among consumers who have a combination of elastic intertemporal substitution and high risk aversion, a mixture that is impossible under time-additive preferences.

There are three periods, periods 0, 1, and 2. A consumer is endowed with w_0 units of consumption goods in period 0, and has access to financial markets to allocate consumption goods between three periods. The consumer invests in risk-free assets in period 0 and in risky assets in period 1. Investment in period 0 yields a fixed return R^f per period, whereas investment in period 1 yields a random return R^x per period, which takes positive values $(r_1, \dots, r_m)$ with probability $p^T = (p_1, \dots, p_m)$, where $p_i = \Pr(R^x = r_i)$.

In period 1, the consumer receives a signal Y, which is correlated with the period 2 realization of R^x. The arrival of such a signal to some extent resolves uncertainty concerning a random return. The signal takes a value of $(y_1, \dots, y_n)$ with probability $q^T = (q_1, \dots, q_n)$, where $q_j = \Pr(Y = y_j)$. The consequent posterior probability distribution is denoted by $\Pi = (\pi_{ij})$, where $\pi_{ij} = \Pr(R^x = r_i | Y = Y_j)$. By construction, $\Pi q = p$. As long as the period 1 signal Y is correlated with the period 2 risky return R^x, a consumer can improve the prediction of R^x based on the realization of Y, and make a better decision in period 2. In this sense, the period 1 signal is referred to as informative; it helps to partially resolve the period 2 uncertainty and improve lifetime utility.[9]

Under the above information structure, we can differentiate between the riskiness of the period 2 investment opportunity and the degree of informativeness of a signal. On the one hand, the variance implied by $p_i = \Pr(R^x = r_i)$ can represent the riskiness from the period 0 perspective (the unconditional variance). On the other hand, the correlation between Y and R^x with fixed $\Pi q = p$ can denote the informativeness of the period 1 signal given the unconditional riskiness of the third period return.

A consumer behaves according to Kreps–Porteus preferences. The most important feature of Kreps–Porteus preferences is that it is possible to determine separately an elasticity of intertemporal substitution (σ) and a degree of relative risk aversion (γ); γ is inversely proportional to σ, or $\sigma\gamma = 1$ under time-additive preferences with constant relative risk aversion.

The consumer maximizes the following objective function:

$$\max_a \left[(w_0 - a)^{\frac{\sigma-1}{\sigma}} + \beta \left(\sum_j q_j J(a, y_j)^{1-\gamma} \right)^{\frac{\sigma-1}{\sigma(1-\gamma)}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

[9] Appendix A offers the exact definition of the informativeness of a signal.

where:

$$J(a, y_j) = \max_x \left[(R^f a - x)^{\frac{\sigma-1}{\sigma}} + \beta \left(\sum_i \pi_{ij} (r_i x)^{1-\gamma} \right)^{\frac{\sigma-1}{\sigma(1-\gamma)}} \right]^{\frac{\sigma}{\sigma-1}} ;$$

a $(0 \leq a \leq w_0)$ and x $(0 \leq x \leq R^f a)$ denote savings in periods 0 and 1, respectively; and $\beta (> 0)$ is a discount factor.

As mentioned above, σ and γ can be chosen independently under Kreps–Porteus preferences. If $\sigma\gamma = 1$ holds, equation (1) reduces to

$$\max_a \left[(w_0 - a)^{1-\gamma} + \beta \left(\sum_j q_j J(a, y_j)^{1-\gamma} \right) \right]^{\frac{1}{1-\gamma}} , \qquad (2)$$

or the standard time-additive utility function.[10]

A consumer, who obtains a signal Y concerning future opportunities for risky investment in period 1, faces two alternatives in period 0: consuming immediately or saving via risk-free assets. The motivation for saving via risk-free assets are: i) smoothing consumption, and ii) waiting for the arrival of informative signals. Given the inability to trade risky assets, only investment in risk-free assets allows the consumer to behave flexibly in response to new information. As the expectation that uncertainty will be resolved grows, consumers may allocate more resources from current consumption to risk-free savings.

As proved in Appendix B, the following proposition holds.[11]

[10] The utility function characterized by equation (2) is called 'time-additive' because the expected payoff $\left(\sum_j q_j J(a, y_j)^{1-\gamma} \right)$ is appended to the current period utility $(w_0 - a)^{1-\gamma}$ in an additive manner. If the current period utility and the expected payoff is linear like equation (2), the utility function is called 'expected.' If they are non-linear like equation (1), it is called 'non-expected.'

[11] A setup of Miyazaki and Saito (2004) is less general than the three-period framework adopted in this paper in that the latter introduces Kreps–Porteus preferences over the three periods, while the former introduces them only between the second and third periods. Miyazaki and Saito (2004) prove that more informative signals raise risk-free savings in period 0 if:

$$\sigma > 1, \ \sigma\gamma \geq 1 \quad \text{or} \quad 0 < \sigma < 1, \ \sigma + \gamma < 2,$$

and more informative signals reduce savings if:

$$\sigma > 1, \ \sigma + \gamma < 2 \quad \text{or} \quad 0 < \sigma < 1, \ \sigma\gamma \geq 1.$$

It should be noted that the conditions in their framework are only sufficient conditions for holding savings as a waiting option motive, whereas our conditions are necessary and sufficient conditions.

Proposition: More informative signals raise risk-free savings in period 0 if and only if:

$$\sigma > 1,\ \sigma + \gamma > 2 \quad \text{or} \quad 0 < \sigma < 1,\ \sigma + \gamma < 2,$$

and more informative signals reduce savings if:

$$\sigma > 1,\ \sigma + \gamma < 2 \quad \text{or} \quad 0 < \sigma < 1,\ \sigma + \gamma > 2.$$

It is under the former condition that a consumer postpones a commitment to current expenditures on consumption goods with more informative signals.

We have several comments about the above proposition. First, the result of Epstein (1980) corresponds to the case in which an informative signal leads to an increase in savings when an elasticity of intertemporal substitution is greater than one ($\sigma > 1$), given that $\sigma\gamma = 1$ under time-additive preferences. In the case analyzed by Epstein, highly elastic intertemporal substitution or low risk aversion plays an essential role in generating saving behavior while waiting for new information.

Second, in addition to σ, we include a degree of relative risk aversion γ separately under Kreps–Porteus preferences. If $\sigma > 1$, then $\sigma + \gamma > 2$ promotes a postponement of consumption commitment when there is an informative signal. As shown in Epstein and Zin (1989) and Weil (1990), $\sigma\gamma > 1$ implies preference for early resolution of uncertainty concerning a consumption sequence.[12] Because a preference for early resolution or $\sigma\gamma > 1$ is a sufficient condition for $\sigma + \gamma > 2$ in the above proposition, we can say that a consumer with a strong preference for early resolution and highly elastic intertemporal substitution always increases savings in order to wait for new information.[13]

Finally, mostly importantly for this paper, our result may be useful in investigating not only saving behavior, but also its consequence on the generation of premiums. Use of the time-additive utility framework often produces the following dilemma. Strong intertemporal substitution ($\sigma > 1$) enhances demand for risk-free assets while waiting for informative signals, but inevitably weakens aversion to risk ($\gamma < 1$), thereby making every asset return close to risk-free rates and narrowing risk premiums substantially. Consequently, strong

[12] A preference for early resolution implies that a consumer prefers the consumption process with early resolution to that with late resolution. As mentioned in footnote 1, early (late) resolution of uncertainty implies that given an unconditional volatility of underlying investment opportunities, a conditional volatility diminishes quickly (slowly) as time passes.

[13] Another interesting result is that even if intertemporal substitution is rather low ($0 < \sigma < 1$), a postponement of consumption commitment occurs if $\sigma + \gamma < 2$. A possible interpretation of this result is that a low degree of intertemporal substitution may reverse the effect on optimal savings. In any case, what affects saving behavior is not the level of risk aversion alone, but a combination of risk aversion and intertemporal substitution.

demand for risk-free assets as waiting options may yield fairly weak effects on asset pricing under time-additive preferences. As shown in the following sections, however, strong demand for risk-free assets generates significant impacts on asset pricing when both intertemporal substitution ($\sigma > 1$) and risk aversion ($\gamma > 1$) are strong. That is, premiums commanded by demand for risk-free assets as waiting options may not be negligible under Kreps–Porteus preferences.

3 A General Equilibrium Framework

In this and the following sections, we explore general equilibrium implications of the model presented in the previous section, in particular asset pricing implications of saving behavior while waiting for new information. Exploiting the merit of the information structure characterized in the previous section, we carefully differentiate the premium generated by exercising waiting options in expectation of the arrival of new information (called waiting-options premiums) from the premium triggered by the riskiness of the period 2 investment opportunity (called risk premiums).

For the above purpose, the three-period model in the previous section is embedded in an overlapping generations (OLG) economy. Each generation is referred to as young, middle-aged, or old. The population of each generation is constant over time and standardized to one. No heterogeneity is present within any generation. On the other hand, while all generations have identical preferences, a particular generation may receive different information.

Each generation has access to financial markets to allocate consumption goods over three periods. Young consumers are endowed with w_0 (> 0) units of goods and in one period can lend or borrow risk-free assets. As in the previous model, young consumers are not allowed to participate in risky asset markets.[14]

Unlike the previous model, middle-aged consumers endowed with $w_1 (> 0)$ units can invest in one-period risky assets as well as in one-period risk-free assets. Short positions are allowed in risk-free assets. The consumers transact in the financial markets in a competitive manner once they participate. One-period returns on risky assets R_t^x are given exogenously, whereas one-period risk-free rates R_t^f are determined endogenously as a result of transactions

[14] With this kind of participation constraint in risky asset markets, waiting-option premiums are expected to emerge in a significant manner. Even if this constraint concerning risky asset investment is replaced by costly transactions or irreversibility, we may obtain similar results about option premiums, but with greater analytical difficulty.

between young and middle-aged consumers.

For the purpose of numerical experiments in the next section, we consider a parsimonious characterization of the informativeness of signals following Jones and Ostroy (1984). According to the information obtained initially by a consumer born at date t, a two-period-ahead return R^x_{t+2} will take a value of r_1 with probability $p_1 = \alpha$ or a value of r_2 with probability $p_2 = 1 - \alpha$ ($r_1 > r_2$). When middle aged, the consumer receives an additional signal Y_{t+1} concerning a one-period ahead risky return R^x_{t+2}. The signal takes a value of y_1 with probability $q_1 = \alpha$ or a value y_2 with probability $q_2 = 1 - \alpha$. The probability of R^x_{t+2} conditional on the interim signal (π_{ij}) is characterized as follows:

$$\begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} \rho + \alpha(1-\rho) & \alpha(1-\rho) \\ 1-\rho-\alpha(1-\rho) & 1-\alpha(1-\rho) \end{bmatrix}. \tag{3}$$

In the above information structure, a parameter $\rho \in [0, 1]$ does not affect the unconditional probability of risky returns at all; Πq is always equal to: $\begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix}$. Thus, a parameter $\rho \in [0, 1]$ represents purely the degree of informativeness of the signal. Indeed, as proved in Appendix C, as ρ approaches one, the signal is more informative. Extreme cases include the perfect resolution of uncertainty offered by the arriving signal when $\rho = 1$, and the absence of resolution when $\rho = 0$. In the next section, we consider both the case in which all generations receive signals in an identical manner and the case in which only a particular generation can receive the interim signal.

Given the above initial endowments and financial opportunities, a representative consumer born at date t maximizes the following problem with respect to an investment plan (risk-free bonds a^t_t and a^t_{t+1}, and risky assets x^t_{t+1}):

$$\max_{a^t_t} \left[(w_0 - a^t_t)^{\frac{\sigma-1}{\sigma}} + \beta \left\{ E_t \left(J(a^t_t, Y_{t+1})^{1-\gamma} \right) \right\}^{\frac{\sigma-1}{\sigma(1-\gamma)}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

where:

$$\begin{aligned} J(a^t_t, Y_{t+1}) \quad = \quad & \max_{a^t_{t+1} x^t_{t+1}} \Big[(w_1 + R^f_t a^t_t - a^t_{t+1} - x^t_{t+1})^{\frac{\sigma-1}{\sigma}} \\ & + \beta \left\{ E_{t+1} \left((R^f_{t+1} a^t_{t+1} + R^x_{t+2} x^t_{t+1})^{1-\gamma} \right) \right\}^{\frac{\sigma-1}{\sigma(1-\gamma)}} \Big]^{\frac{\sigma}{\sigma-1}}, \end{aligned} \tag{5}$$

and E_t is the conditional expectation operator based on the information available at date t.

An equilibrium risk-free rate is determined endogenously by the lending–borrowing process between young and middle-aged consumers. Using dynamic programming techniques, we can derive the optimal asset demand a_t^t, a_{t+1}^t, and x_{t+1}^t as:

$$\begin{aligned} a_t^t &= f^t(\Omega_t), \\ a_{t+1}^t &= g^t(a_t^t, R_t^f, R_{t+1}^f, Y_{t+1}), \\ x_{t+1}^t &= h^t(a_t^t, R_t^f, R_{t+1}^f, Y_{t+1}), \end{aligned}$$

where the information set Ω_t is recursively defined as $\Omega_t = \Big\{\Omega_{t-1},\ x_{t-1}^{t-2},\ a_{t-1}^{t-2},\ a_{t-1}^{t-1},\ R_t^f,\ R_t^x,\ Y_t\Big\}$. See Appendix D for more detailed descriptions of f^i, g^i, and h^i. Then, an equilibrium risk-free rate R_t^f is determined such that:

$$a_t^t + a_t^{t-1} = 0. \tag{6}$$

4 A Numerical Investigation

This section presents the numerical results of several experiments in order to demonstrate how an equilibrium risk-free rate is influenced by both the riskiness of investment opportunities and the informativeness of arriving signals within the framework constructed in the previous section. In this section, decreases in risk-free rates driven by risk-averse behavior are called *risk premiums*, whereas decrements in risk-free rates caused by informativeness of signals are called *waiting-options premiums*.

More specifically, on the one hand, risk premiums correspond to the extent to which risk-free rates are driven by mean-preserving spreads of risky returns R_{t+2}^x from the perspective of a young consumer born at date t. On the other hand, waiting-options premiums are defined as the extent to which risk-free rates change owing to degrees of informativeness of signals or changes in ρ. Note that in this OLG model, *ex ante* excess returns (premiums) can be defined as the difference between exogenously given unconditional means of risky returns and equilibrium risk-free rates, because risk-free rates are determined in equilibrium before the interim signal is realized.

We consider the following cases. In the first case, hereafter referred to as **Case 0**, there is no informative content in signals at all, and $\rho = 0$ for all generations. In this case, only risk premiums can be examined through the effects of mean-preserving spreads of risky returns on risk-free rates. In contrast with Case 0, the second case (**Case 1**) takes the informativeness of signals into consideration. That is, all generations with identical preferences receive the interim signal when they are middle aged. In principle, Case 1 can reveal how waiting-options premiums are determined within a general equilibrium framework. As shown below, in Case 1, demand for risk-free assets does indeed emerge because of increases in ρ, but such demand is not reflected in risk-free rates in a significant manner. A major reason for this is that an informative signal enhances the demand of young consumers for risk-free assets, but it also promotes a shift from safe to risky assets among middle-aged consumers and lowers their demand for safe assets. Consequently, the impacts of waiting options on risk-free rates are canceled out by decreases in safe-asset demand from middle-aged consumers and are negligible.

We prepare an additional case to highlight waiting-options effects on risk-free rates. In the third case (**Case 2**), only a particular generation can receive the interim signal; $\rho > 0$ for a particular generation and $\rho = 0$ for the other generations. In other words, intergenerational heterogeneity is introduced into the parameter ρ. The numerical procedures of the above three cases are described briefly in Appendix E.

For quantitative experiments, we choose admissible values of parameters β, r_1, r_2, α, w_0, w_1, σ, γ, and ρ. The choice of parameters here is motivated not by attempts to mimic a real economy, but by efforts to explore the qualitative implications of the above OLG model. β is set to be $1/1.02$ throughout the experiments. Both r_1 and r_2 are chosen such that the unconditional mean is equal to 1.1. Our numerical procedure begins with the setup where $r_1 = 1.2$, $r_2 = 1.0$, and $\alpha = 0.5$ ($E(R^x) = 1.1$). In terms of endowment, w_0 and w_1 are assumed to be 30 and 100, respectively. Such an assumption concerning initial endowments would promote young consumption instead of young savings.

With respect to preference parameters, the elasticity of intertemporal substitution σ takes values between $1/3$ and 8, whereas γ changes from 1 to 8. Accordingly, the choice of preference parameters includes a combination of elastic intertemporal substitution and high risk aversion ($\sigma\gamma > 1$) and a combination of inelastic intertemporal substitution and low risk aversion ($\sigma + \gamma < 2$). Note that $\sigma\gamma > 1$ ($\sigma + \gamma < 2$) implies $\sigma + \gamma > 2$ ($\sigma\gamma < 1$). The degree of informativeness of signals ρ takes a value of either 0.0 or 0.8. In most examples, therefore, waiting-options premiums are defined as the differences in risk-free rates between cases where $\rho = 0.0$ and where $\rho = 0.8$.

4.1 Case 0: no informative content in signals

Table 1 summarizes the numerical results of Case 0, where $\rho = 0$ for all generations. A steady-state equilibrium emerges as an immediate consequence of fixed risky investment opportunities. As mentioned before, a risk premium is defined as $E(R^x) - R^f$, and decreases in risk-free rates result in increases in risk premiums.

Table 1: The numerical result of Case 0 with $r_1 = 1.2$ and $r_2 = 1.0$

σ	γ	$R^f(\%)$	$a_0 = -a_1$	x_1
1/3	1	9.380	-13.206	28.125
1/3	2	8.764	-13.252	28.148
1/3	3	8.156	-13.297	28.171
1/3	4	7.561	-13.341	28.195
1/3	5	6.982	-13.384	28.219
1/3	6	6.424	-13.425	28.243
1/3	7	5.889	-13.465	28.268
1/3	8	5.381	-13.502	28.293
1	1	9.326	-11.294	32.098
1	2	8.665	-11.483	31.844
1	3	8.022	-11.670	31.595
1	4	7.401	-11.852	31.352
1	5	6.806	-12.028	31.117
1	6	6.240	-12.198	30.892
1	7	5.704	-12.360	30.677
1	8	5.200	-12.514	30.474
3	1	9.197	-5.818	43.882
3	2	8.428	-6.479	42.676
3	3	7.702	-7.121	41.510
3	4	7.022	-7.737	40.392
3	5	6.390	-8.324	39.330
3	6	5.807	-8.880	38.328
3	7	5.271	-9.402	37.387
3	8	4.780	-9.890	36.506
8	1	9.008	6.053	71.903
8	2	8.078	4.356	68.318
8	3	7.224	2.679	64.811
8	4	6.453	1.056	61.441
8	5	5.764	-0.490	58.250
8	6	5.153	-1.942	55.265
8	7	4.615	-3.292	52.498
8	8	4.142	-4.538	49.949

Beginning with the assumption that $r_1 = 1.2$, $r_2 = 1.0$, and $\alpha = 0.5$, when a degree of risk aversion γ increases, given elasticity of intertemporal substitution σ, middle-aged consumers increase their risk-free investments but decrease their risky investments. This means that middle-aged consumers with greater risk aversion shift funds from risky to safe assets. Consequently, the risk-free rate declines.

When σ increases, given γ, young consumers reduce their risk-free borrowing, while middle-aged consumers increase their risky investment. A young consumer with large elasticity of intertemporal substitution tends to allocate more to future consumption, given that risk-free rates are higher than time-preference rates (which are equal to 2% throughout the numerical exercises). Such consumption allocation in turn raises demand for risk-free bonds from middle-aged consumers through wealth effects. Increases in demand for safe assets from both young and middle-aged consumers jointly contribute to decreases in equilibrium risk-free rates. However, the effect of σ on risk-free rates is not as strong as that of γ.

Nevertheless, we conjecture that the above monotonic depressing (increasing) effect of risk aversion on risk-free rates (risk premiums) may be weakened when risk-free rates are below time-preference rates as a result of the introduction of large risks. When the riskiness of the future investment opportunity is extremely large, young consumers may consume immediately instead of transferring resources to the future. Such a tendency may be more pronounced for those with both stronger intertemporal substitution and larger risk aversion, as these individuals tend to be more interested in choosing the timing of consumption *and* are more averse to future consumption volatilities.

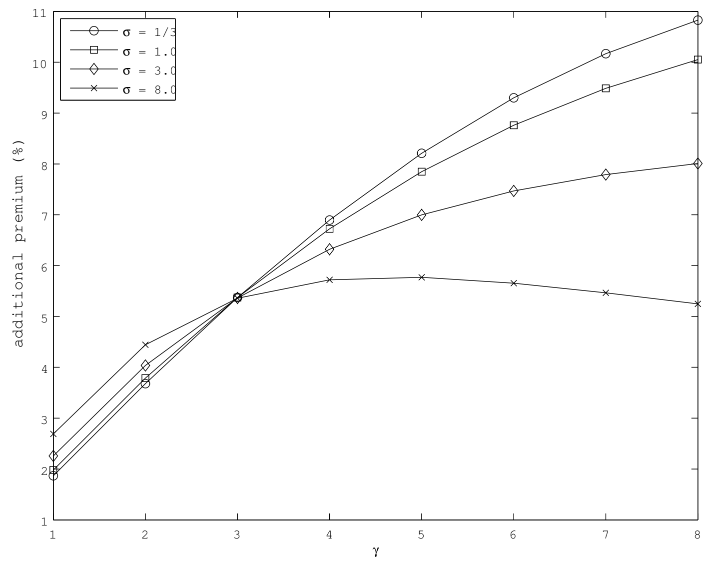
Figure 1 raises riskiness to $r_1 = 1.3$ and $r_2 = 0.9$ by mean-preserving spreads and compares it with $r_1 = 1.2$ and $r_2 = 1.0$. According to this figure, additional risk premiums are still monotonically increasing in risk aversion for those with relatively weak intertemporal substitution.[15] For those with $\sigma = 8$, however, additional risk premiums are decreasing when γ is above four. This kind of finding is not available from a time-additive utility framework where it is impossible to increase σ and γ simultaneously.

4.2 Case 1: with informative signals

Unlike Case 0, Case 1, where all generations receive the interim message ($\rho > 0$), generates a stationary Markov equilibrium. That is, risk-free rates change over time depending on which state of y_1 or y_2 is realized, and investment and

[15] In Figure 1, additional risk premiums happen to be similar for various values of σ at $\gamma = 3$, but they are still different and distinct in rigorous terms.

Figure 1: Additional risk premiums due to more volatile risky returns (Case 0)



(1) The above figure plots differences in risk premiums between under $r_1 = 1.3$ and $r_2 = 0.9$ and under $r_1 = 1.2$ and $r_2 = 1.0$ in Case 0.

consumption plans are influenced by the movement of risk-free rates. (See Appendix E for a more detailed characterization of this stationary Markov equilibrium.)

Table 2 reports the unconditional means of risk-free rates and investment plans under $\rho = 0.8$.[16] In addition, the last column of Table 2 presents waiting-options premiums, defined as differences in risk-free rates between such rates under $\rho = 0.8$ (reported in the third column of Table 3) and under $\rho = 0.0$ (reported in the third column of Table 1). Figure 2 depicts how demand functions for risk-free assets from young consumers change as the informativeness of signals becomes greater under $\sigma = \gamma = 3$.

As shown in Section 2, large elasticities of intertemporal substitution ($\sigma >$

[16] For this calculation, 5200 random variables of the interim message and risky returns are generated, and given this fixed random seed, equilibrium risk-free rates and investment plans are derived numerically. The unconditional means of these variables are computed after dropping the first 200 observations.

Table 2: The numerical result of Case 1 with $r_1 = 1.2$ and $r_2 = 1.0$

σ	γ	$R^f(\%)$	$a_0 = -a_1$	x_1	waiting-options premium (%)
1/3	1	9.776	-13.204	28.146	-0.396
1/3	2	9.550	-13.209	28.172	-0.786
1/3	3	9.321	-13.214	28.198	-1.165
1/3	4	9.086	-13.220	28.223	-1.525
1/3	5	8.845	-13.226	28.248	-1.863
1/3	6	8.597	-13.233	28.272	-2.173
1/3	7	8.340	-13.242	28.295	-2.450
1/3	8	8.073	-13.251	28.317	-2.693
1	1	9.755	-11.337	32.098	-0.429
1	2	9.507	-11.406	32.004	-0.841
1	3	9.252	-11.476	31.909	-1.230
1	4	8.990	-11.548	31.811	-1.589
1	5	8.719	-11.622	31.711	-1.913
1	6	8.438	-11.697	31.608	-2.199
1	7	8.148	-11.776	31.501	-2.444
1	8	7.847	-11.857	31.391	-2.647
3	1	9.709	-5.972	44.049	-0.512
3	2	9.408	-6.252	43.543	-0.979
3	3	9.095	-6.537	43.026	-1.393
3	4	8.768	-6.830	42.495	-1.747
3	5	8.428	-7.130	41.949	-2.038
3	6	8.074	-7.439	41.387	-2.267
3	7	7.708	-7.756	40.810	-2.438
3	8	7.334	-8.080	40.221	-2.554
8	1	9.649	5.096	71.556	-0.641
8	2	9.284	4.394	70.137	-1.206
8	3	8.900	3.680	68.684	-1.675
8	4	8.494	2.945	67.183	-2.041
8	5	8.066	2.182	65.620	-2.303
8	6	7.620	1.389	63.991	-2.467
8	7	7.160	0.563	62.296	-2.545
8	8	6.693	-0.291	60.541	-2.551

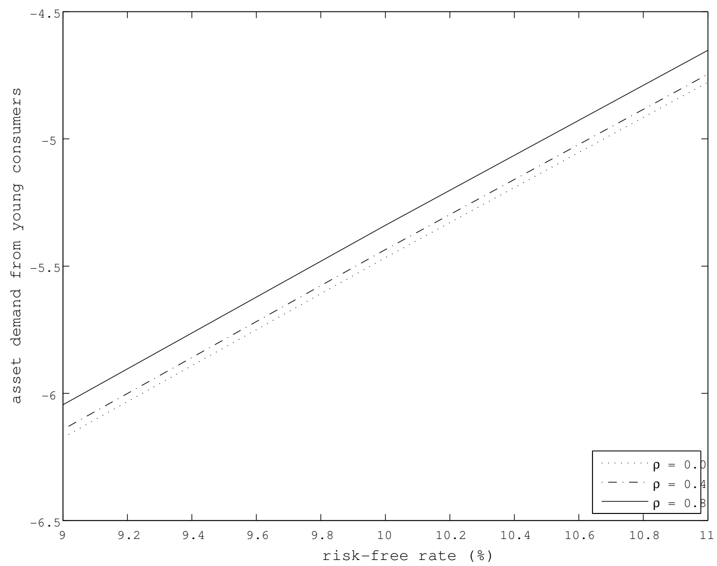
1) as well as high degrees of relative risk aversion ($\sigma\gamma > 1$) jointly contribute to increases in demand for risk-free assets in the current setup. Figure 2 demonstrates that demand for risk-free assets from young consumers is boosted as ρ becomes closer to one. Nevertheless, Table 2 documents negative waiting-options premiums. In other words, although demand for risk-free assets is generated, such demand is not reflected directly in the equilibrium behavior of risk-free rates. A major reason for the above asymmetry between demand and risk-free rates is that a larger ρ raises demand for risk-free assets from the young consumers of the current generation, but it promotes a shift from risk-free assets to risky assets among the middle-aged consumers of the previous generation, as a result of the arrival of informative signals. In other words, stronger demand for risk-free assets from young consumers is largely canceled out by weaker demand for risk-free assets from middle-aged consumers. Therefore, waiting-options effects on risk-free rates are not observed clearly in the numerical result of Case 1. As previously suggested, Case 2 introduces intergenerational heterogeneity in order to highlight waiting-options impacts on risk-free rates.

4.3 Case 2: intergenerational heterogeneity in ρ

In Case 2, only one particular generation can receive the interim message, whereas the other generations do not. More concretely, only the generation born at date T receives the interim message Y_{T+1} with $\rho_T = 0.8$. On the other hand, preference parameters σ and γ are common among all generations. For simplicity, it is assumed that generation $t < T$ does not know that generation T receives the interim signal, and that generation $t > T$ does not expect the arrival of any interim signal at all. Based on this setup, demand for risk-free assets from middle-aged consumers of generation $T-1$ is completely independent of the informativeness of signals. Accordingly, demand for risk-free assets from young consumers of generation T may be translated almost directly into equilibrium risk-free rates. Note that *ex ante* excess returns (premiums) are still defined as the deviation of unconditional means of risky returns from risk-free rates because risk-free rates are determined in equilibrium before the arrival of the interim signal. One possible interpretation of this setup is that a particular generation happens to face resolvable uncertainty.

Table 3 presents the numerical result of date T risk-free rates and investment plans (R^f_T, $a^t_T(= -a^{T-1}_T)$, and x^{T-1}_T) under $\rho_t = 0.8$. In addition, the last column of Table 3 reports waiting-options premiums, which are defined as differences in risk-free rates between values under $\rho = 0.8$ and under $\rho = 0.0$. Figure 3 depicts waiting-options premiums for various values of σ

Figure 2: Demand functions for safe assets from young consumers and resolution of uncertainty (Case 1)



(1) The above figure plots demand functions for safe assets from young consumers with $\rho = 0.0$, 0.4, and 0.8 under $\sigma = 3$ and $\gamma = 3$.

when $1 \leq \gamma \leq 8$, whereas Figure 4 plots waiting-options premiums when $0 < \gamma < 1$. Comparing these waiting-options premiums with the demand for safe assets from young consumers (a_0) in Table 1 (also reported in parentheses in the fourth column of Table 3), it is possible to explore whether demand for risk-free assets is indeed promoted by the informativeness of signals.

The numerical results are summarized as follows. First, if the elasticity of intertemporal substitution σ is equal to one, then demand for safe assets from young consumers is completely independent of the informativeness of signals, and there are no waiting-options premiums.

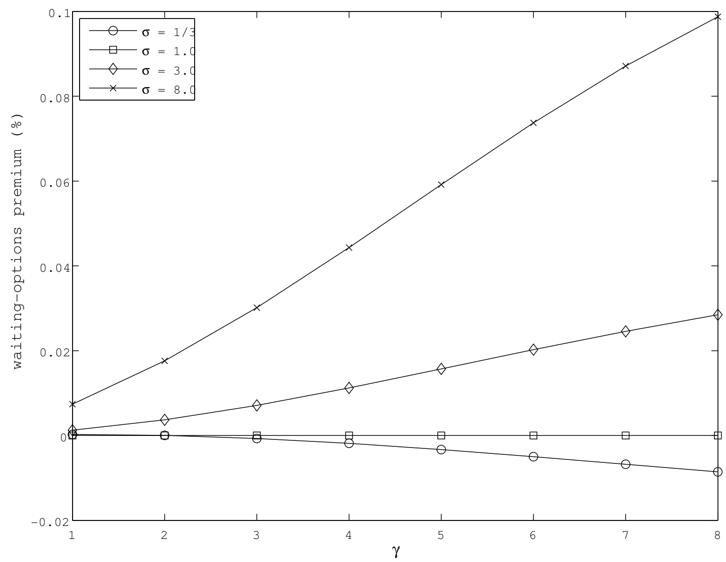
Second, when σ is greater than one, and γ is also higher ($\sigma + \gamma > 2$), then demand for risk-free assets emerges and positive waiting-options premiums are generated. In particular, as shown in Figure 3, waiting-options premiums increase with the degree of relative risk aversion, γ, given $\sigma > 1$. The second feature is consistent with the proposition in Section 2 that demand for risk-free assets emerges when $\sigma > 1$ and $\sigma + \gamma > 2$. Consistent with Epstein's

Table 3: The numerical result of Case 2 with $r_1 = 1.2$ and $r_2 = 1.0$

σ	γ	$R_T^f(\%)$	$a_T^T = -a_T^{T-1}$	x_T^{T-1}	waiting-options premium (%)
1/3	1	9.380	-13.196(-13.206)	28.135	0.000
1/3	2	8.764	-13.251(-13.252)	28.149	0.000
1/3	3	8.157	-13.306(-13.297)	28.162	-0.001
1/3	4	7.562	-13.360(-13.341)	28.176	-0.002
1/3	5	6.985	-13.412(-13.384)	28.191	-0.003
1/3	6	6.428	-13.462(-13.425)	28.206	-0.004
1/3	7	5.895	-13.510(-13.465)	28.222	-0.006
1/3	8	5.388	-13.555(-13.502)	28.239	-0.007
1	1	9.326	-11.294(-11.294)	32.098	0.000
1	2	8.665	-11.483(-11.483)	31.844	0.000
1	3	8.022	-11.670(-11.670)	31.595	0.000
1	4	7.401	-11.852(-11.852)	31.352	0.000
1	5	6.806	-12.028(-12.028)	31.117	0.000
1	6	6.240	-12.198(-12.198)	30.892	0.000
1	7	5.704	-12.360(-12.360)	30.677	0.000
1	8	5.200	-12.514(-12.514)	30.474	0.000
3	1	9.195	-5.752(-5.818)	43.948	0.001
3	2	8.425	-6.382(-6.479)	42.773	0.003
3	3	7.695	-6.994(-7.121)	41.636	0.007
3	4	7.011	-7.584(-7.737)	40.545	0.010
3	5	6.376	-8.147(-8.324)	39.507	0.015
3	6	5.788	-8.681(-8.880)	38.525	0.019
3	7	5.248	-9.185(-9.402)	37.602	0.023
3	8	4.754	-9.658(-9.890)	36.736	0.026
8	1	9.000	6.625(6.053)	72.476	0.008
8	2	8.060	5.036(4.356)	69.000	0.018
8	3	7.193	3.466(2.679)	65.600	0.031
8	4	6.407	1.942(1.056)	62.329	0.045
8	5	5.704	0.485(-0.490)	59.225	0.060
8	6	5.079	-0.892(-1.942)	56.313	0.074
8	7	4.529	-2.182(-3.292)	53.601	0.086
8	8	4.046	-3.384(-4.538)	51.092	0.097

(1) The numbers in parentheses in the fourth column are the risk-free savings of young consumers reported in Table 1.

Figure 3: Waiting-options premiums based on $\rho = 0.0$ versus $\rho = 0.8$ (Case 2)



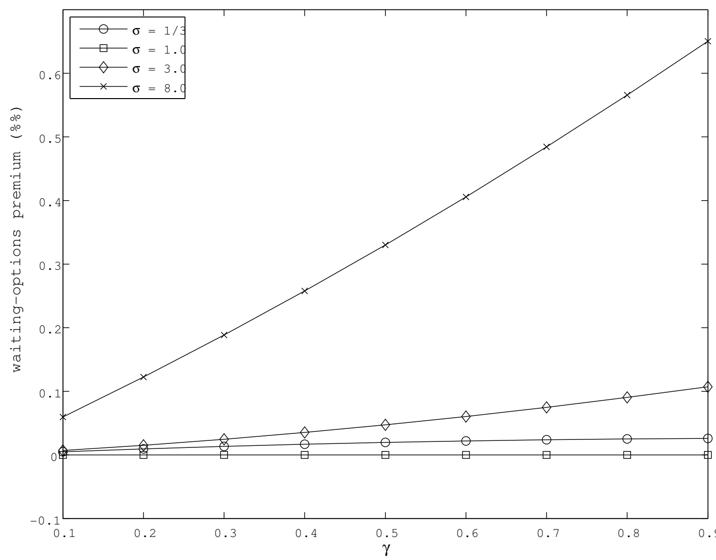
(1) The above figure plots differences in risk-free rates between with $\rho = 0.8$ and with $\rho = 0.0$ under $r_1 = 1.2$ and $r_2 = 1.0$.

(1980) result, even CRRA preferences yield positive waiting-options premiums provided σ is larger than one ($\sigma = 3, \gamma = 1/3$ and $\sigma = 8, \gamma = 1/8$ in Figure 4). However, as discussed in Section 2, only tiny premiums are generated under low values of γ.

Third, even if σ is less than one, waiting-options premiums, although extremely small, are yielded as long as both σ and γ are small (see the case where $\sigma = \frac{1}{3}$ in Figures 3 and 4). The third observation is again consistent with the proposition that another condition for demand for risk-free assets is that $0 < \sigma < 1$ and $\sigma + \gamma < 2$.

Inferring from the result of Case 0, we can reasonably expect that large risks associated with investment opportunities may have an additional impact on risk-free rates or premiums. Given an extremely risky investment, young consumers with strong intertemporal substitution and large risk aversion may consume instead of saving, thereby canceling out demand for risk-free assets. Suppose that generation T faces values of $r_1 = 1.3$ and $r_2 = 0.9$, with $\rho = \sqrt{1/2}$, and that the other generations experience values of $r_1 = 1.2$ and

Figure 4: Waiting-options premiums based on $\rho = 0.0$ versus $\rho = 0.8$ with $\gamma < 1$ (Case 2)



(1) The above figure plots differences in risk-free rates between with $\rho = 0.8$ and with $\rho = 0.0$ under $r_1 = 1.2$ and $r_2 = 1.0$.

$r_2 = 1.0$, with $\rho = 0$. The parameter $\rho = \sqrt{1/2}$ is chosen such that the conditional volatility for generation T is exactly equal to the unconditional volatility for the other generations without any interim message in terms of average absolute deviations. One possible interpretation of this setup is that a particular generation happens to face large, but resolvable, uncertainty.

Table 4 presents the numerical results of the above case. In addition, Figure 5 plots waiting-options premiums, which are defined as the deviations from the risk-free rate of Case 0, with $r_1 = 1.2$ and $r_2 = 1.0$. As Figure 5 demonstrates, when elasticity of intertemporal substitution is large ($\sigma = 8$), demand for risk-free assets is canceled out largely by a disincentive for young consumers to save when γ is beyond four and premiums (risk-free rates) are decreasing (increasing) in risk aversion.

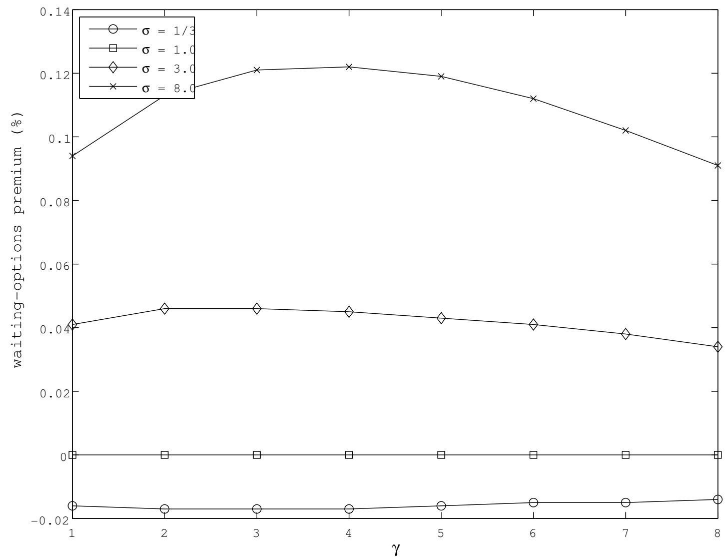
In sum, young consumers with a combination of elastic intertemporal substitution and high risk aversion, generate demand for risk-free assets, and such demand is reflected directly in equilibrium risk-free rates. Extreme riskiness

Table 4: The numerical result of the case with large but resolvable uncertainty for generation T ($r_1 = 1.3$ and $r_2 = 0.9$ with $\rho = \sqrt{1/2}$ for generation T versus $r_1 = 1.2$ and $r_2 = 1.0$ with $\rho = 0.0$ for all generations)

σ	γ	$R_T^f(\%)$	$a_T^T = -a_T^{T-1}$	x_T^{T-1}	waiting-options premium (%)
1/3	1	9.396	-13.918(-13.206)	27.412	-0.016(0.000)
1/3	2	8.781	-13.641(-13.252)	27.757	-0.017(0.000)
1/3	3	8.173	-13.563(-13.297)	27.905	-0.017(-0.001)
1/3	4	7.577	-13.542(-13.341)	27.993	-0.017(-0.002)
1/3	5	6.998	-13.545(-13.384)	28.057	-0.016(-0.003)
1/3	6	6.440	-13.560(-13.425)	28.108	-0.015(-0.004)
1/3	7	5.904	-13.580(-13.465)	28.152	-0.015(-0.006)
1/3	8	5.395	-13.602(-13.502)	28.192	-0.014(-0.007)
1	1	9.326	-11.294(-11.294)	32.098	0.000(0.000)
1	2	8.665	-11.483(-11.483)	31.844	0.000(0.000)
1	3	8.022	-11.670(-11.670)	31.595	0.000(0.000)
1	4	7.401	-11.852(-11.852)	31.352	0.000(0.000)
1	5	6.806	-12.028(-12.028)	31.117	0.000(0.000)
1	6	6.240	-12.198(-12.198)	30.892	0.000(0.000)
1	7	5.704	-12.360(-12.360)	30.677	0.000(0.000)
1	8	5.200	-12.514(-12.514)	30.474	0.000(0.000)
3	1	9.155	-3.543(-5.818)	46.156	0.041(0.001)
3	2	8.383	-5.212(-6.479)	43.940	0.046(0.003)
3	3	7.656	-6.245(-7.121)	42.383	0.046(0.007)
3	4	6.977	-7.071(-7.737)	41.055	0.045(0.010)
3	5	6.347	-7.792(-8.324)	39.859	0.043(0.015)
3	6	5.766	-8.443(-8.880)	38.761	0.041(0.019)
3	7	5.233	-9.038(-9.402)	37.747	0.038(0.023)
3	8	4.746	-9.584(-9.890)	36.809	0.034(0.026)
8	1	8.914	12.977(6.053)	78.849	0.094(0.008)
8	2	7.965	8.561(4.356)	72.541	0.113(0.018)
8	3	7.104	5.737(2.679)	67.881	0.121(0.031)
8	4	6.331	3.451(1.056)	63.843	0.122(0.045)
8	5	5.645	1.453(-0.490)	60.194	0.119(0.060)
8	6	5.041	-0.343(-1.942)	56.861	0.112(0.074)
8	7	4.513	-1.969(-3.292)	53.813	0.102(0.086)
8	8	4.051	-3.446(-4.538)	51.031	0.091(0.097)

(1) The numbers in parentheses in the fourth column are the risk-free savings of the young consumers reported in Table 1.
(2) The numbers in parentheses in the sixth column are the waiting-options premiums reported in Table 3.

Figure 5: Waiting-options premiums with large, but resolvable uncertainty



(1) The above figure plots differences in risk-free rates between under $r_1 = 1.3$ and $r_2 = 0.9$ with $\rho = \sqrt{1/2}$ for generation I, and under $r_1 = 1.2$ and $r_2 = 1.0$ with $\rho = 0.0$ for all generations.

of investment opportunities, on the other hand, dampens demand for risk-free assets to some extent, and tends to raise risk-free rates for those with both strong intertemporal substitution and high risk aversion.

5 Conclusion

In this paper, we have presented an overlapping generations framework where an *ex ante* excess return over a risk-free rate can be divided into a risk premium component and a waiting-options premium component. In this framework, an incentive to postpone consumption until an informative signal arrives triggers demand for risk-free assets. Such demand may result in decreases in risk-free rates or increases in waiting-options premiums. By nature, waiting-options premiums are fairly different from risk premiums driven by the riskiness of investment opportunities.

Our numerical examples have shown that consumers with elastic intertemporal substitution as well as high risk aversion, generate stronger demand for risk-free assets, thereby resulting in positive waiting-options premiums. It should be emphasized that the generation of sizable waiting options premiums is possible only under Kreps–Porteus preferences, because a combination of elastic intertemporal substitution and high risk aversion is not feasible at all under time-additive preferences. In this regard, our investigation sheds light on another advantage of Kreps–Porteus preferences in dynamic asset pricing models.

Appendix

In this appendix, we explain the definition of the degree of informativeness (Appendix A), the proof of the proposition presented in Section 2 (Appendix B), the proof of ρ in matrix (3) as a measure of informativeness (Appendix C), a description of optimal decision functions discussed in Section 3 (Appendix D), and the numerical procedure adopted in Section 4 (Appendix E).

A. Definition of the degree of informativeness

The following definition of informativeness was originally proposed by Marschak and Miyasawa (1968), and subsequently discussed by Epstein (1980), Jones and Ostroy (1984), and others. In addition to a signal Y defined in Section 2, we consider another signal Y', which takes a value of $(y'_1, \ldots, y'_n)$ with probability $q'^T = (q'_1, \ldots, q'_n)$, where $q'_j = \Pr(Y' = y'_j)$. Π' is defined such that the prior distribution is fixed or $\Pi' q' = p$. The signal Y' is called *more informative* than Y if:

$$\sum_j q'_j \Phi(\pi'_j) \geq \sum_j q_j \Phi(\pi_j), \tag{7}$$

for any convex function Φ, where π_j and π'_j are the j-th columns of Π and Π', respectively. Clearly, the reverse inequality of equation (7) holds if Φ is concave.

B. Proof of the proposition

We solve the above maximization problem (1) backwards to prove the proposition. The first-order condition with respect to x leads to:

$$x = R^f a (1 + \beta^{-\sigma} (\sum_i \pi_{ij} r_i^{1-\gamma})^{\frac{1-\sigma}{1-\gamma}})^{-1}.$$

Then, using the above equation and the first-order condition with respect to x_0, we obtain:

$$w_0 - x_0 = \beta^{-\sigma}(R^f)^{1-\sigma} x_0 \left[\sum_j q_j \zeta(\pi_j)\right]^{\frac{1-\sigma}{\gamma-1}}, \tag{8}$$

where:

$$\zeta(\pi_j) = (1 + \beta^{\sigma}(\sum_i \pi_{ij} r_i^{1-\gamma})^{\frac{\sigma-1}{1-\gamma}})^{\frac{1-\gamma}{\sigma-1}}.$$

Equation (8) enables investigation of the effect of the resolution of uncertainty on period 0 savings or on the choice between consumption commitment and liquid assets. Suppose that Y' is more informative than Y. Let x_0^* (x_0^{**}) be the optimum savings based on Y (Y'). From the definition of the degree to which Y is informative, based on inequality (7), we obtain two statements:

1. If $\frac{1-\sigma}{\gamma-1} > 0$ and $\zeta(\pi_j)$ for each j is convex (concave), then $x_0^{**} \geq x_0^*$ ($x_0^{**} \leq x_0^*$).
2. If $\frac{1-\sigma}{\gamma-1} < 0$ and $\zeta(\pi_j)$ for each j is convex (concave), then $x_0^{**} \leq x_0^*$ ($x_0^{**} \geq x_0^*$).

We will demonstrate a condition for the convexity (concavity) of $\zeta(\pi_j)$. The first derivative with respect to π_{ij} is:

$$\frac{\partial \zeta(\pi_j)}{\partial \pi_{ij}} = r_i^{1-\gamma}\beta^{\sigma}[1 + \beta^{\sigma}(\sum_i \pi_{ij} r_i^{1-\gamma})^{\frac{\sigma-1}{1-\gamma}}]^{\frac{1-\gamma}{\sigma-1}-1}(\sum_i \pi_{ij} r_i^{1-\gamma})^{\frac{\sigma-1}{1-\gamma}-1}.$$

The second derivative with respect to π_{ij} and π_{kj} is:

$$\begin{aligned}\frac{\partial^2 \zeta(\pi_j)}{\partial \pi_{ij}\partial \pi_{kj}} &= r_i^{1-\gamma} r_k^{1-\gamma}\frac{\sigma+\gamma-2}{1-\gamma}\beta^{\sigma}[1 + \beta^{\sigma}(\sum_i \pi_{ij} r_i^{1-\gamma})^{\frac{\sigma-1}{1-\gamma}}]^{\frac{1-\gamma}{\sigma-1}-2}(\sum_i \pi_{ij} r_i^{1-\gamma})^{\frac{\sigma-1}{1-\gamma}-2} \\ &\equiv r_i^{1-\gamma} r_k^{1-\gamma}\omega.\end{aligned}$$

The Hessian matrix is thus defined as:

$$\omega \begin{bmatrix} r_1^{1-\gamma} \\ \vdots \\ r_n^{1-\gamma} \end{bmatrix} [r_1^{1-\gamma} \cdots r_n^{1-\gamma}].$$

The Hessian is positive definite if and only if $\omega > 0$. A necessary and sufficient condition for the convexity of $\zeta(\pi_j)$ is $\frac{\sigma+\gamma-2}{1-\gamma} > 0$. Note that $r_i^{1-\gamma}$, π_{ij}, and β^{σ} are all positive for all i. A necessary and sufficient condition for the concavity of $\zeta(\pi_j)$ is obtained analogously: $\frac{\sigma+\gamma-2}{1-\gamma} < 0$.

Using these conditions, the statements can be rewritten as:

1. If $\frac{1-\sigma}{\gamma-1} > 0$ and $\frac{\sigma+\gamma-2}{1-\gamma} > 0$ ($\frac{\sigma+\gamma-2}{1-\gamma} < 0$), then $x_0^{**} \geq x_0^*$ ($x_0^{**} \leq x_0^*$).
2. If $\frac{1-\sigma}{\gamma-1} < 0$ and $\frac{\sigma+\gamma-2}{1-\gamma} > 0$ ($\frac{\sigma+\gamma-2}{1-\gamma} < 0$), then $x_0^{**} \leq x_0^*$ ($x_0^{**} \geq x_0^*$).

In other words:

1. If $(\sigma > 1,\ \sigma+\gamma > 2)$ or $(0 < \sigma < 1,\ \sigma+\gamma < 2)$, then $x_0^{**} \geq x_0^*$.
2. If $(\sigma > 1,\ \sigma+\gamma < 2)$ or $(0 < \sigma < 1,\ \sigma+\gamma > 2)$, then $x_0^{**} \leq x_0^*$.

C. Proof of ρ in matrix (3) as a measure of informativeness

Given matrix (3), we have $m = n = 2$, $p^T = q^T = (\alpha, 1-\alpha)$, $0 < \alpha < 1$, $\Pi = [\pi_1\ \pi_2]$, $\pi_1^T = (\rho+\alpha(1-\rho), 1-\rho-\alpha(1-\rho))$, $\pi_2^T = (\alpha(1-\rho), 1-\alpha(1-\rho))$. Consider a case of $0 \leq \rho < 1$. For any convex function Φ, we define:

$$W(\alpha,\rho) \ \equiv \ \sum q_j \Phi(\pi_j) = \alpha\Phi(\pi_1) + (1-\alpha)\Phi(\pi_2).$$

If $\tau^T = (1,-1)$, then $W(\alpha,\rho) = \alpha\Phi(q+\rho(1-\alpha)\tau) + (1-\alpha)\Phi(q-\rho\alpha\tau)$. By the nature of convex functions, we obtain:

$$W(\alpha,\rho) \geq \Phi[\alpha(q+\rho(1-\alpha)\tau) + (1-\alpha)(q-\rho\alpha\tau)] = \Phi(q).$$

When $\rho = 0$, $W(\alpha, 0) = \Phi(q)$ holds.

Choose ρ' such that $0 < \rho' < \rho$. If η is defined as ρ'/ρ, then $0 < \eta < 1$, and:

$$q + \rho'(1-\alpha)\tau = (1-\eta)q + \eta(q+\rho(1-\alpha)\tau).$$

Again by the nature of convex functions:

$$\Phi(q+\rho'(1-\alpha)\tau) \leq (1-\eta)\Phi(q) + \eta\Phi(q+\rho(1-\alpha)\tau).$$

Similarly, we obtain $\Phi(q-\rho'\alpha\tau) \leq (1-\eta)\Phi(q) + \eta\Phi(q-\rho\alpha\tau)$. That is, we have:

$$\begin{aligned} W(\alpha,\rho') &= \alpha\Phi(q+\rho'(1-\alpha)\tau) + (1-\alpha)\Phi(q-\rho'\alpha\tau) \\ &\leq (1-\eta)\Phi(q) + \eta\{\alpha\Phi(q+\rho(1-\alpha)\tau) + (1-\alpha)\Phi(q-\rho\alpha\tau)\} \\ &\leq (1-\eta)W(\alpha,\rho) + \eta W(\alpha,\rho) = W(\alpha,\rho). \end{aligned}$$

Then, as ρ approaches one, a signal is more informative.

D. Optimal decision functions

This subsection briefly explains how to derive optimal decision functions from the maximization problem (4) in Section 3. As usual, these decision functions are solved by backward induction. Consider a consumer born at date t. When middle aged, the consumer solves the maximization problem to find the value function $J(a_t^t, Y_{t+1})$ (5). The first-order conditions with respect to a_{t+1}^t and x_{t+1}^t yield:

$$\begin{aligned} a_{t+1}^t &= (w_1 + R_t^f a_t^t) \cdot D_0 \cdot (r_1 D_2 - r_2 D_1), \text{ and} \\ x_{t+1}^t &= (w_1 + R_t^f a_t^t) \cdot D_0 \cdot R_{t+1}^f \cdot (D_1 - D_2), \end{aligned}$$

where:

$$\begin{aligned} D_0(R_{t+1}^f, Y_{t+1}) &= [D_1(R_{t+1}^f - r_2) + D_2(r_1 - R_{t+1}^f) + R_{t+1}^f(r_1 - r_2)]^{-1}, \\ D_1(R_{t+1}^f, Y_{t+1}) &= \tilde{D}_0 \cdot \tilde{D}_1 \cdot [\pi(Y_{t+1})\tilde{D}_1^{1-\gamma} + (1-\pi(Y_{t+1}))\tilde{D}_2^{1-\gamma}]^{\frac{\sigma\gamma-1}{1-\gamma}}, \\ D_2(R_{t+1}^f, Y_{t+1}) &= \tilde{D}_0 \cdot \tilde{D}_2 \cdot [\pi(Y_{t+1})\tilde{D}_1^{1-\gamma} + (1-\pi(Y_{t+1}))\tilde{D}_2^{1-\gamma}]^{\frac{\sigma\gamma-1}{1-\gamma}}, \\ \tilde{D}_0 &= \left(\beta R_{t+1}^f (r_1 - r_2)\right)^{\sigma}, \\ \tilde{D}_1 &= \left(\frac{\pi(Y_{t+1})}{R_{t+1}^f - r_2}\right)^{\frac{1}{\gamma}}, \\ \tilde{D}_2 &= \left(\frac{1-\pi(Y_{t+1})}{r_1 - R_{t+1}^f}\right)^{\frac{1}{\gamma}}, \\ \pi(Y_{t+1}) &= \begin{cases} \rho + \alpha(1-\rho) & \text{if} \quad Y_{t+1} = y_1 \\ \alpha(1-\rho) & \text{if} \quad Y_{t+1} = y_2 \end{cases}. \end{aligned}$$

Given a_t^t, R_t^f, R_{t+1}^f and Y_{t+1}, demand functions for safe and risky assets, $g(a_t^t, R_t^f, R_{t+1}^f, Y_{t+1})$ and $h(a_t^t, R_t^f, R_{t+1}^f, Y_{t+1})$, are characterized analytically. When the intergenerational heterogeneity is introduced, an upper subscript t is added, such as: $g^t(a_t^t, R_t^f, R_{t+1}^f, Y_{t+1})$ and $h^t(a_t^t, R_t^f, R_{t+1}^f, Y_{t+1})$.

Substituting these decision functions into equation (4) leads to the middle-aged value function:

$$J(a_t^t, Y_{t+1}) = (w_1 + R_t^f a_t^t) V(R_{t+1}^f, Y_{t+1}),$$

where:

$$V = D_0 \cdot R_{t+1}^f \cdot (r_1 - r_2) \left[1 + \beta \left\{\pi(Y_{t+1}) D_1^{1-\gamma} + (1-\pi(Y_{t+1})) D_2^{1-\gamma}\right\}^{\frac{\sigma-1}{\sigma(1-\gamma)}}\right]^{\frac{\sigma}{\sigma-1}}.$$

Substituting the above value function $J(a_t^t, Y_{t+1})$ into equation (5), we obtain the following objective function maximized by a young consumer of generation t:

$$\left[(w_0 - a_t^t)^{\frac{\sigma-1}{\sigma}} + \beta(w_1 + R_t^f a_t^t)^{\frac{\sigma-1}{\sigma}} E_t \left\{V(R_{t+1}^f, Y_{t+1})^{1-\gamma}\right\}^{\frac{\sigma-1}{\sigma(1-\gamma)}}\right]^{\frac{\sigma}{\sigma-1}}.$$

Note that Ω_t contains a_t^t and R_t^f, but not Y_{t+1} and R_{t+1}^f. The first-order condition with respect to a_t^t yields:

$$a_t^t = \frac{w_0\left[\beta R_t^f E_t\left\{V(R_{t+1}^f, Y_{t+1})^{1-\gamma}\right\}^{\frac{\sigma-1}{\sigma(1-\gamma)}}\right]^{\sigma} - w_1}{\left[\beta R_t^f E_t\left\{V(R_{t+1}^f, Y_{t+1})^{1-\gamma}\right\}^{\frac{\sigma-1}{\sigma(1-\gamma)}}\right]^{\sigma} + R_t^f}.$$

The decision function of a_t^t, denoted as f or f^t, depends both on R_t^f and on the conditional expectation of $V(R_{t+1}^f, Y_{t+1})^{1-\gamma}$.

In the following cases, the decision functions f, g, or h may be expressed in a simpler manner. First, if $\sigma = 1$, then $E_t\left\{V(R_{t+1}^f, Y_{t+1})^{1-\gamma}\right\}^{\frac{\sigma-1}{\sigma(1-\gamma)}}$ reduces to $1+\beta$, and f is accordingly equal to $\frac{\beta(1+\beta)w_0}{1+\beta+\beta^2} - \frac{w_1}{R_t^f(1+\beta+\beta^2)}$. That is, even if the interim message is expected to arrive, the young consumer's demand for safe assets never changes in this case. In addition, γ is irrelevant in determining the young saving–consumption decision. Second, if $\rho = 0$, then the decision functions g and h do not depend on Y_{t+1}. As a result, f is explained solely by R_t^f and the conditional expectation of R_{t+1}^f. Third, when all generations have identical preferences and R^f is constant over time, as in Case 0, the decision functions are described as $a_0 = f(R^f)$, $a_1 = g(a_0, R^f)$, and $x_1 = h(a_0, R^f)$. Fourth, when all generations have identical preferences and R_{t+1}^f follows a stationary transition function of R_t^f and Y_{t+1}, as in Case 1, then the decision functions are described as $a_0 = f(R_t^f)$, $a_1 = g(a_0, R_t^f, R_{t+1}^f, Y_{t+1})$, and $x_1 = h(a_0, R_t^f, R_{t+1}^f, Y_{t+1})$. These properties of the decision functions are used in calculating equilibrium risk-free rates in the numerical experiment.

E. Numerical procedure

This subsection briefly explains how to obtain the numerical results reported in Section 4. A basic procedure is as follows.

1. Guess a sequence of risk-free rates.

2. Given the above guess, solve the dynamic optimization problem in the manner described in the previous subsection.

3. Update a new sequence of the risk-free rate using the numerically derived decision functions and the market-clearing condition (6). Use a bisection method to find the equilibrium risk-free rate that satisfies equation (6).

4. Iterate the above steps until a sequence of risk-free rates converges.

In terms of the market-clearing condition (6), $f(R^f) + g(f(R^f), R^f) = 0$ in Case 0 where a steady-state equilibrium is obtained, whereas $f(R_t^f) + g(f(R_{t-1}^f), R_{t-1}^f, R_t^f, Y_t) = 0$ in Case 1 where a stationary Markov equilibrium emerges.

In Case 2, the equilibrium is neither stationary nor in a steady state because of the intergenerational heterogeneity. Any generation where $t < T$ follows the same decision functions as in Case 0. For generations where $t > T$, an equilibrium is influenced, which results in the state of Y_{T+1} being realized, and a_T^T can be denoted as $f^t(R_T, Y_{T+1})$ for $t > T$. Given these equilibrium conditions, R_T^f is determined such that the sum of a_T^{T-1} and a_T^T is equal to zero.

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