

# Poverty measurement under income risk

Armutsmessung bei Einkommensrisiken

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*Abstract:* Poverty indices are usually calculated on the basis of (cross section) income data from the past. As past income is fixed and future income is uncertain such measurement does not reflect the *ex ante* risk of falling into poverty. This paper presents refinements of commonly used poverty measures to account for income risk. It is shown that the standard headcount ratio underestimates poverty in societies with moderate poverty if income is risky. The Foster, Greer and Thorbecke measure always underestimates poverty if income risk is neglected.

*Zusammenfassung:* Die Berechnung traditioneller Armutsmaße beruht zumeist auf ex post Einkommensdaten. Dabei wird implizit von einem sicheren Einkommen der Individuen ausgegangen. Aus einer *ex ante* Perspektive ist das Einkommen jedoch mit Risiko behaftet, und damit besteht ein Armutsrisiko auch für diejenigen, deren erwartetes Einkommen oberhalb der Armutsgrenze liegt. Der vorliegende Beitrag stellt modifizierte Armutsmaße vor, die Einkommensrisiken Rechnung tragen. Für Gesellschaften mit moderater Armut kann gezeigt werden, dass eine traditionelle Messung der Armutsquote das Ausmaß der Armut unterschätzt, wenn Einkommensrisiken unberücksichtigt bleiben. Ein Foster-Greer-Thorbecke-Maß weist bei Einkommensrisiken immer eine Unterschätzung der Armut auf.

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# 1 Introduction

Poverty is widespread in developing countries. What makes things worse is uncertainty of income. In industrialised countries the poor are a small minority, however, a much larger share of the population is at risk of falling into poverty. Although there is an increasing concern for poverty risks (e.g. Leibfried and Leisering 1999), the measurement of poverty under income risk has received very little attention. Most studies in poverty measurement use household income data to calculate one or more of various poverty measures such as the head count ratio, the income gap ratio, or Sen's (1976) poverty index. With the use of data from a previous period each person appears to receive a fixed income. This reflects an *ex post* perspective. However, a person with sufficient income *ex post* may face a poverty risk *ex ante*. This risk is concealed in the data from previous periods.

The aim of this paper is to develop risk-adjusted poverty measures. Such measures will be useful to assess policies which are concerned not just with the prevalence of poverty but also with poverty risks. From this perspective a policy which stabilises incomes may achieve a reduction of poverty risk even if it does not reduce the number of the poor. A risk-adjusted poverty measure would reflect such achievement.

In recent years a number of studies have been looking at the impact of income risk on savings and consumption (e.g. Skinner 1988, Guiso et al. 1996, Blundell and Preston 1998, Blundell and Stoker 1999). The question of how social insurance tackles income risk has been addressed by Haveman and Wolfe (1985), Bird (1995) and Bird and Hagstrom (1999). Bane and Ellwood (1986) and Stevens (1994, 1999) have estimated probabilities of poor households to escape from poverty. Burgess et al. (2000) estimate income risks due to demographic and labour market factors. But only few studies have explored links between risk and poverty measurement. Kakwani (1995) examines a class of poverty measures when the poverty line is uncertain. He motivates his study by a missing consensus about the poverty line. However, assuming the poverty line reflects minimum needs, his results apply when needs are uncertain. In this paper we consider the case of income risk.<sup>1</sup> A poor person's income falls short of the poverty line. Hence, income risk may generate poverty risk. How income risk affects poverty measurement has been studied by Ravallion (1988) and Bigman (1993). Ravallion (1988) assumes that all individuals are exposed to the same risk. Bigman (1993) uses the income distribution across population to represent risks.<sup>2</sup> In this paper we do not use these

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<sup>1</sup> For ease of presentation, we use income as an indicator for poverty. Other possible indicators are, for example, wealth, consumption, standard of living or Sen's (1985) concept of capabilities.

<sup>2</sup> Bigman (1993) studies food security rather than poverty. He presents a modification of Sen's (1976) poverty index which has been developed into the Aggregate Household Food Security Index (cf. FAO 1994; FAO 1997).

assumptions but rather consider independent income risks without imposing a restriction on the risk distribution. Distributions are only specified for illustrative purposes, but do not affect the structure of the risk-adjusted indices we suggest. The paper examines how individual poverty risks can be aggregated into a single risk-adjusted poverty index which measures expected poverty. We then compare *expected* poverty under income risk with poverty as measured by the corresponding standard poverty index which does not account for risk. In general, we find that standard measurement underestimates (expected) poverty if income is risky. Hence, if poverty risk is considered a relevant issue, an assessment of poverty related social policies cannot rely on standard poverty indices. In this case, a more appropriate analysis is supported by the risk-adjusted indices proposed in section 3 of this paper.

In what follows we develop refinements of three widely used poverty measures: (i) the proportion of poor people in society, commonly called the headcount ratio  $H$ ; (ii) the relative shortfall of income of the poor with regard to the poverty line, called the income gap ratio  $I$ ; and (iii) a class of poverty measures suggested by Foster, Greer and Thorbecke (1984)  $F$ . Poverty measures  $H$  and  $I$  are chosen because they are by far the most widely used. In his seminal work Sen (1976) has noted that these measures are distribution-insensitive. This has triggered a vast literature on alternative poverty measures which has been surveyed by Foster (1984), Seidl (1988) and Zheng (1997). Zheng (1997, 143) has listed the properties of the various poverty measures suggested in the literature.  $F$  is among those measures which satisfy all properties which can reasonably be desired of a poverty measure. Moreover, Ebert and Moyes (2002) have recently provided an axiomatic characterisation of the  $F$ -class of poverty measures. This warrants the effort to provide a risk-adjusted version of this measure.

The paper is organised as follows. The next section introduces the notation and the measures  $H$ ,  $I$  and  $F$  as a point of reference. Section 3 provides the necessary refinements of the three poverty measures when income is risky. Section 4 explores the effects of income risk on poverty as measured by risk-adjusted indices  $H$  and  $F$  and compares standard poverty measurement with risk-adjusted measurement. Section 5 concludes.

## 2 Definitions and notation

Poverty measurement involves two problems: the identification of the poor and the aggregation of information about the poor (Sen 1976). Most studies focus on income as an indicator to identify the poor. For ease of presentation I follow this tradition, but the analysis is more general and applies to other poverty indicators as well.

We define a minimum income requirement, the poverty line  $z$ , such that all individuals  $i$  who earn income  $y_i < z$  are said to be poor. This implicitly assumes that minimum income requirements, or needs, are the same for everyone.<sup>3</sup> Consider a society  $N$  with  $n$  members. The income distribution is  $(y_1, \dots, y_n)$ . Without loss of generality, we assume  $y_1 \leq \dots \leq y_n$ . A poverty measure is a normalised index  $P \in [0, 1]$  attached to each  $(y_1, \dots, y_n; z)$ . Furthermore, denote the set of the poor  $M$  and the number of the poor  $m$ ; thus  $y_m < z \leq y_{m+1}$ .

In order to introduce the standard approach to poverty measurement we assume here that each person's income is given. Income risk is introduced in the next section. Starting point of analysis are three poverty measures which are most widely used. We consider the headcount ratio  $H$ , the income gap ratio  $I$ , and a measure suggested by Foster, Greer and Thorbecke (1984)  $F$ , which are defined below.

The simplest and most prominent poverty measure is the ratio of the poor in the entire population, commonly called headcount ratio

$$H = \frac{m}{n}. \quad (2.1)$$

$H$  does not capture the intensity of poverty. An index which measures how poor the poor are is the income gap ratio  $I$ . The income gap of individual  $i$  is  $z - y_i$ . Let  $\bar{y} = \frac{1}{m} \sum_{i=1}^m y_i$  be the average income of the poor. Then the relative shortfall of the average income of the poor, called income gap ratio, is defined as

$$I = \frac{z - \bar{y}}{z}. \quad (2.2)$$

The measure suggested by Foster, Greer and Thorbecke (1984) gives an increasing weight to increasing individual income gap ratios.

$$F = \frac{1}{n} \sum_{i=1}^m \left( \frac{z - y_i}{z} \right)^\alpha, \quad (2.3)$$

where  $\alpha \geq 0$  is a constant. Note that for  $\alpha = 0$   $F = H$  and for  $\alpha = 1$   $F = HI$ . For  $\alpha > 2$   $F$  is known to satisfy various desirable properties.<sup>4</sup>

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<sup>3</sup> If the relevant unit is the household, differences in needs may stem from differences in household composition. For individuals disability, age or other characteristics may cause differences in needs. As people differ in needs an individual poverty line  $z_i$  which may be different for each individual  $i$  could be introduced. An analogous problem has been discussed in the context of the measurement of undernourishment; see Sukhatme (1961), Anand and Harris (1992), Kakwani (1992) and Gabbert and Weikard (2001) and the references therein.

<sup>4</sup> See Foster, Greer and Thorbecke (1984) and Zheng (1997, 150 f) for a full account of its properties.

### 3 Introducing income risk to poverty measurement

The application of the poverty indices introduced in section 2 usually requires cross section income data. Such data provide a snapshot picture of poverty at a certain point in time. One way to extend the analysis is the use of panel data which complements the picture with information on income mobility. How poverty indices can account for the persistence or transience of poverty has been explored in earlier work (Weikard 2000). The case of income risk is different and must be clearly distinguished from income changes over time. Seasonal unemployment, for example, leads to a fall and subsequent rise in income. However, insofar as the temporal employment pattern can be foreseen, there is no risk involved. On the other hand, even if there is no change in income over time, income may not be risk free.

To see the importance of risk for poverty measurement, consider an individual  $i$  with income  $y_i \geq z$ . Individual  $i$  does not count as poor from an *ex post* perspective. However, *ex ante*, if  $y_i$  is a random variable,  $i$  may face a risk of being poor. Poverty measures using information about incomes in past periods may understate (or overstate) the impact on poverty when future incomes are uncertain. Hence, the purpose of this section is to construct expected poverty measures that account for income risk.

An *ex ante* poverty measure can be constructed in a two step procedure. First, we derive an expected poverty index for an individual on the basis of the probability distribution of  $i$ 's income expressed by a probability density function  $f_i(y_i)$ . Second, given the poverty status of each individual, we construct an aggregate poverty index for a society of  $n$  individuals. The aggregation proposed in what follows rests upon the assumption that individual income distributions are independent of each other. Hence, the indices are not suitable to capture macroeconomic risks.

The probability that  $i$ 's income will fall below the poverty line, i.e. the probability that  $i$  is poor, is given by

$$\pi_i = \int_0^z f_i(y_i) dy_i . \quad (3.1)$$

Given independent probability distributions of income for all individuals, we can calculate the expected head count ratio as

$$\hat{H} = \frac{1}{n} \sum_{i=1}^n \pi_i . \quad (3.2)$$

$\hat{H}$  is a straightforward generalisation of the standard headcount ratio  $H$ . If probabilities  $\pi_i$  are restricted to be either 0 or 1, then  $\hat{H}$  is equivalent to  $H$  (see equation 2.1).

Similarly, to derive the expected income gap ratio we again first look at a single individual. Person  $i$  is poor with probability  $\pi_i$ . Conditional on being poor her expected income is

$$\hat{y}_i = \frac{1}{\pi_i} \int_0^z y_i f_i(y_i) dy_i. \quad (3.3)$$

Note that in this case  $\pi_i > 0$ . The expected income gap ratio of individual  $i$  if  $i$  is poor is given by

$$\hat{I}_i = \frac{z - \hat{y}_i}{z}. \quad (3.4)$$

To see how the expected income gap ratio for the society can be calculated consider any given set  $M \subseteq N$  of poor people. Their average income is

$$\hat{y}_M = \frac{1}{m} \sum_{i \in M} \hat{y}_i. \quad (3.5)$$

In this case the expected income gap ratio becomes

$$\hat{I}_M = \frac{z - \hat{y}_M}{z} = \frac{1}{m} \sum_{i \in M} \frac{z - \hat{y}_i}{z}. \quad (3.6)$$

Next, notice that the probability that a particular set  $M$  is the group of the poor is  $\prod_{i \in M} \pi_i \cdot \prod_{j \in N-M} (1 - \pi_j) \equiv \mu_M$ . The expected income gap ratio is a probability weighted sum of the income gap ratio for all possible subsets of the poor,

$$\hat{I} = \sum_{M \subseteq N} \mu_M \hat{I}_M. \quad (3.7)$$

The Foster, Greer and Thorbecke index  $F$  sums up weighted individual income gap ratios over the range of income below  $z$ . A risk-adjusted  $F$ -index for individual  $i$  is given by

$$\hat{F}_i = \int_0^z \left( \frac{z - y_i}{z} \right)^\alpha f_i(y_i) dy_i. \quad (3.8)$$

In this case the aggregation across individuals is straightforward. The expected  $F$ -index is

$$\hat{F} = \frac{1}{n} \sum_{i=1}^n \hat{F}_i. \quad (3.9)$$

## 4 On the underestimation of poverty when income risk is neglected

This section explores some of the impacts of income risk on poverty. Using the measures derived in section 3 we can identify conditions under which the neglect of income uncertainty in common analyses leads to underestimation or overestimation of poverty. First, section 4.1 illustrates income risk effects on poverty measurement for a single individual. Section 4.2 explores estimation errors that occur if income risk is neglected and it presents a sufficient condition for the underestimation of the head count ratio of poverty. Section 4.3 analyses the underestimation effect of the Foster, Greer and Thorbecke index. It also introduces the notion of a certainty equivalent income which is the income that, if received with certainty, would lead to the same level of poverty (as measured by the  $F$ -index) as the risky income. Section 4.4 provides empirical results on the size of under- or overestimation when risk is neglected for a sample of developing countries.

### 4.1 A simple illustration

To illustrate the impact of risk we compare two situations, A and B. The two situations are the same for all but one individual  $i$ . In situation A each person receives her income with certainty. In situation B individual  $i$  receives a risky income. Assume that  $i$ 's income is low  $y_i^l$  with probability  $\pi_i$ , or high  $y_i^h$  with probability  $1 - \pi_i$ . Denote  $i$ 's expected income  $y_i^e = \pi_i y_i^l + (1 - \pi_i) y_i^h$ . Assume also that  $i$ 's income in situation A is equal to  $y_i^e$ . Hence, B is derived from A by a mean preserving spread.

There are four cases:

- (i)  $y_i^l > z$ . In this case there is no poverty risk for  $i$ .  $i$ 's income risk does not affect poverty measurement.
- (ii)  $y_i^e \geq z$  and  $y_i^l < z$ . Although her expected income is above the poverty line,  $i$  faces a poverty risk in situation B. Thus, the head count ratio  $\hat{H}$  is increased by  $\pi_i/n$  as compared to situation A where  $i$  receives  $y_i^e$  with certainty.

To study the effect of risk on the expected poverty gap ratio  $\hat{I}$ , note that  $\hat{y}_i = y_i^l$ . Moving from situation A to B,  $i$  joins the group of the poor with probability  $\pi_i$ . The effect on the income gap ratio is ambiguous since  $\hat{I}_i$  may be larger or smaller than the average income gap ratio of the rest of the population  $\hat{I}_{-i}$ . Thus the income gap ratio for the society may rise or fall. The ambiguity stems from the fact that poverty index  $I$  violates a strong monotonicity axiom:  $I$  may increase if a poor person receives additional income and thereby leaves the group of the poor; cf. Seidl (1988, 91).

The effect of income risk on the  $\hat{F}$  index is unambiguous. Moving from situation A to B increases  $\hat{I}_i$ . Since in situation B  $\pi_i > 0$ , this will also increase  $\hat{F}$ .

(iii)  $y_i^e < z$  and  $y_i^h \geq z$ . The analysis of this case is similar to case (ii). Moving from situation A to B, we find a decrease in  $\hat{H}$ , because  $i$ 's probability to be poor is now less than 1. Again the effect on the poverty gap ratio  $\hat{I}$  is ambiguous.  $\hat{F}$  shows an increase in poverty as we move from A to B. The  $F$ -index gives an increasing marginal weight to the individual income gap. Thus the chance to climb out of poverty if  $y_i^h$  is received, is overcompensated by the downside risk to receive  $y_i^l$  (see Figure 4.1).

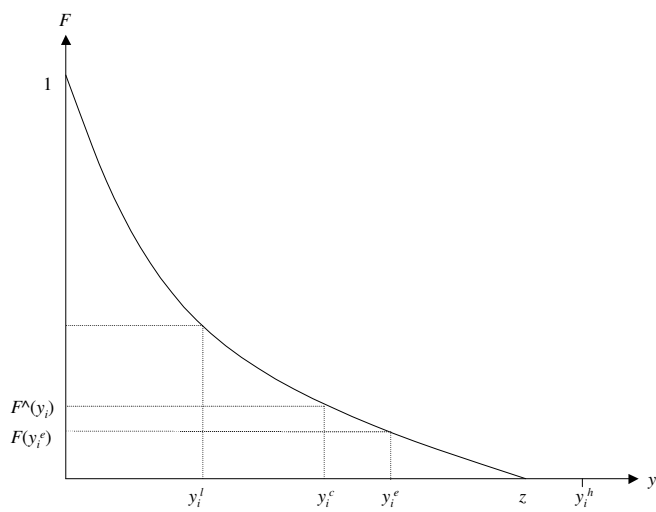


Figure 4.1: Certainty equivalent income for the Foster-Greer-Thorbecke measure

(iv)  $y_i^h < z$ . Finally, we consider an uncertain income of a poor individual  $i$ , where  $i$  remains to be poor even if she receives the high income  $y_i^h$ . In this case there is no effect of risk on the measures  $\hat{H}$  and  $\hat{I}$ . As in case (iii)  $\hat{F}$  shows an increase in poverty as we move from A to B. If  $y_i^e$  is received with certainty the  $F$ -index is lower than the probability weighted average of the  $F$ -index for the risky income. This can be seen from Figure 4.1.  $\hat{F}$  for the risky income corresponds to an income  $y_i^e$  which is the certainty equivalent income of situation B (see figure 4.1).  $\hat{F}$  implicitly captures individual poverty risk aversion.

A risk-adjusted aggregate poverty measure draws together the effects described in the four cases for all individuals. Income risk of individuals in category (i) and (iv) are irrelevant for  $\hat{H}$  and  $\hat{I}$ . The  $\hat{F}$ -index is sensitive towards income risks of the poor. Increasing risk, a mean preserving spread, leads to increasing poverty.



#### 4.2 Estimation errors of the ratio of the poor

Going beyond an illustration of the impact of income risk of a single individual on expected poverty, this section considers aggregate effects when everyone's income is risky. We compare poverty measurement with and without risk. To be more precise, assuming individual income is a random variable, we calculate the estimation error if the measurement of the ratio of the poor is based on expected income instead of applying the risk-adjusted measure which captures the full probability distribution.

From (3.1) and (3.2) we obtain the risk-adjusted ratio of the poor,

$$\hat{H} = \frac{1}{n} \sum_{i=1}^n \int_0^z f_i(y_i) dy_i. \quad (4.1)$$

As before  $i$ 's expected income is denoted  $y_i^e$ . Without loss of generality, we assign index numbers such that  $y_1^e \leq \dots \leq y_m^e < z \leq y_{m+1}^e \leq \dots \leq y_n^e$ . If risk is neglected and expected income is used to measure the ratio of the poor,  $H = m/n$ . The difference between  $\hat{H}$  and  $H$  is due to Type I and Type II statistical classification errors. The Type I error occurs if  $y_i^e < z$  when, using standard analysis,  $i$  counts as poor while there is some positive probability that  $i$  is not poor. This gives rise to an overestimation of expected poverty. The Type II error occurs if  $y_i^e \geq z$  when  $i$  does not count as poor while there is some positive probability that  $i$  is poor. This gives rise to an underestimation of expected poverty.

The Type I overestimation error can be expressed by

$$E_I = \frac{1}{n} \sum_{i=1}^m \left( 1 - \int_0^z f_i(y_i) dy_i \right). \quad (4.2)$$

The Type II underestimation error can be expressed by

$$E_{II} = \frac{1}{n} \sum_{i=m+1}^n \left( \int_0^z f_i(y_i) dy_i \right). \quad (4.3)$$

It holds that

$$\hat{H} = H - E_I + E_{II}. \quad (4.4)$$

Whether the use of  $H$  instead of  $\hat{H}$  under- or overestimates expected poverty depends on the distribution of risks and the distribution of expected income in society. For further analysis we move from a discrete to a continuous distribution of expected income across population. We assume that  $y^e$  is lognormally distributed with density function  $g = g(y^e)$ . Lognormal distributions are widely used to approximate income distributions; cf. e.g. Lambert (1993). Concerning the distribution of risk we assume that

$y_i$  is drawn from an arbitrary symmetric distribution  $D$  with mean  $y_i^e$  and variance  $\sigma^2$ . In this case, the following sufficient condition for underestimation of poverty holds:

*In the presence of income risks where  $y_i$  is drawn from  $D(y_i^e, \sigma^2)$  for all  $i$ , if the distribution  $g(y^e)$  is lognormal, then  $H$  underestimates the expected ratio of the poor  $\hat{H}$  in a society with moderate poverty, i.e. when  $z$  is to the left of the mode of the distribution  $g(y^e)$ .*

To see why the above condition holds, consider the expected income level  $y_i^e = z - \delta$ . At this income level the overestimation error is a fraction  $\int_z^\infty f_i(y_i) dy_i$ . By symmetry of the distribution  $f$  there is an underestimation error of equal size for the expected income level  $y_j^e = z + \delta$ . Figure 4.2 shows over- and underestimation ratios for a triangular distribution  $f$  as equally sized areas  $a$  and  $b$ , respectively. With moderate poverty (when  $z$  is to the left of the mode of the distribution  $g$ ), for every  $\delta > 0$  it holds that  $g(z - \delta) < g(z + \delta)$ . Hence the aggregate underestimation error always dominates the aggregate overestimation error. However with severe poverty, the overestimation error would dominate.

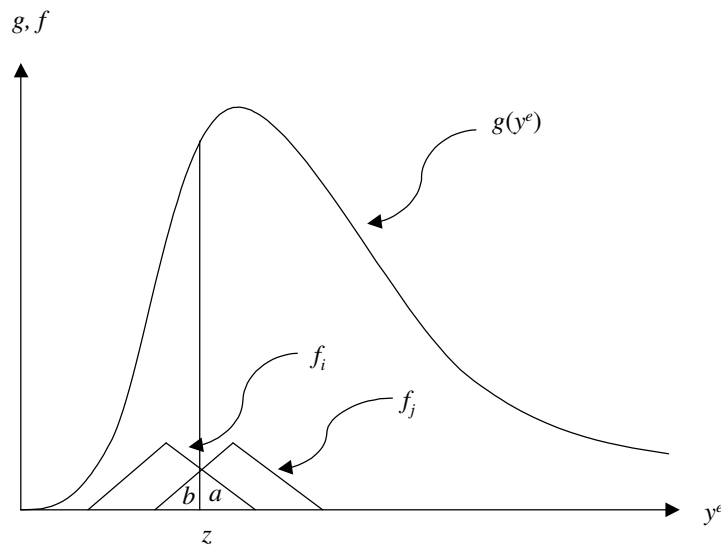


Figure 4.2: Over- and underestimation of the ratio of the poor

### 4.3 Certainty equivalent income

To further examine the effects of risk on poverty it is important to go beyond a simple headcount ratio. Hence we examine the effect of income risk on the Foster-Greer-Thorbecke measure  $F$  in somewhat greater detail. For illustration, we assume that income risk is described by a uniform distribution  $f$  on the interval  $[\lambda y^e, (2 - \lambda)y^e]$  with mean  $y_i^e$ , where  $0 < \lambda \leq 1$ . Hence, in the worst possible case  $i$  receives a fraction  $\lambda$  of

her expected income. Then, using equation (3.8), the risk adjusted measure  $\hat{F}_i(y^e)$  for an individual who receives expected income  $y^e < z/\lambda$  can be specified as follows:

$$\hat{F}_i(y^e) = \frac{1}{2(1-\lambda)y^e} \int_{\lambda y^e}^{\text{Min}[(2-\lambda)y^e, z]} \left( \frac{z - y_i}{z} \right)^\alpha dy_i. \quad (4.7)$$

Those who receive  $y^e \geq z/\lambda$  do not face any poverty risk.

A comparison between a standard measure  $F_i$  and a risk adjusted measure  $\hat{F}_i$  is shown in figure 4.3.

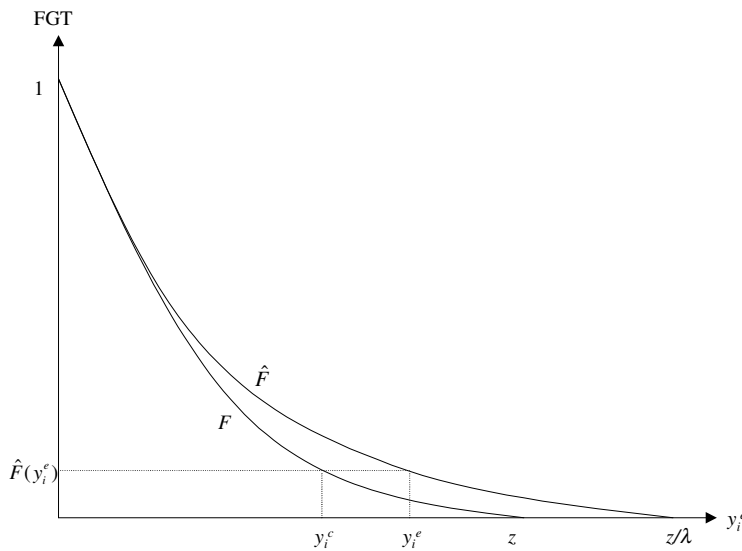


Figure 4.3: Underestimation with a Foster-Greer-Thorbecke measure

$\hat{F}_i$  gives higher values than  $F_i$ . For very low expected income the effect is small because the risk (variance of  $f$ ) is small. While  $F_i$  is zero for all expected incomes larger than  $z$ ,  $\hat{F}_i$  is strictly positive between  $z$  and  $z/\lambda$ . Underestimation of poverty according to a Foster-Greer-Thorbecke measure is severest for individuals who receive an expected income close to the poverty line. Figure 4.3 shows that a risky income with expected income  $y_i^e$  implies the same degree of poverty as a riskless income  $y_i^c$ , hence,  $y_i^c$  is the certainty equivalent income. It can be used to compute a risk-adjusted Foster-Greer-Thorbecke measure for the society,

$$\hat{F} = \frac{1}{n} \sum_{i=1}^n F_i(y_i^c). \quad (4.8)$$

#### 4.4 Empirical assessment of the impact of income risk

In order to further illustrate the impact and empirical relevance of risk adjustments in poverty measurement this section looks at data for a sample of developing countries. We compare measure  $H$  with the corresponding risk-adjusted measure for different levels of risk. We assume that the data reported reflect individuals' expected income. Income risk is introduced by assuming that  $i$ 's income is, as before, drawn from a uniform distribution on the interval  $[\lambda y^e, (2-\lambda)y^e]$ , where  $0 < \lambda \leq 1$ . A lower value of  $\lambda$  corresponds to a higher level of risk. On the bases of these assumptions one can calculate the probability to be poor for each level of expected income  $y^e$ ,

$$\pi(y^e) = \int_0^z f_i(y_i) dy_i = \frac{1}{2(1-\lambda)y^e} \int_{\lambda y^e}^z dy_i = \frac{z - \lambda y^e}{2(1-\lambda)y^e}. \quad (4.9)$$

The expected head count ratio is

$$\hat{H} = \int_0^{\infty} \pi(y^e) g(y^e) dy^e. \quad (4.10)$$

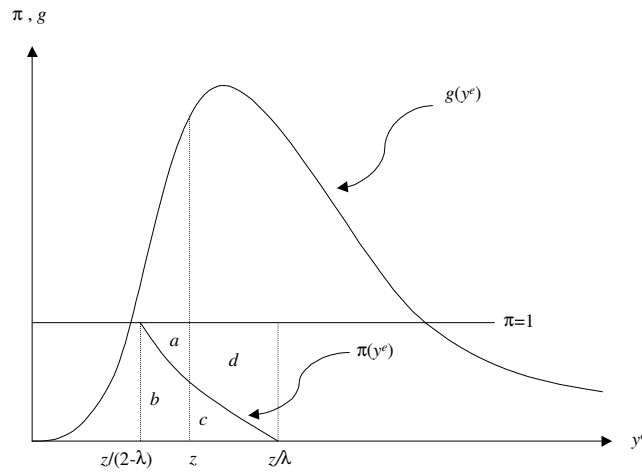


Figure 4.4: Calculation of risk-adjusted poverty measures

$\pi(y^e)$  given in (4.9) has a hyperbolic form as depicted in figure 4.4. The figure shows the areas of over- and underestimation for a lognormal distribution of expected income. Of all people in the range of expected income  $[\frac{z}{2-\lambda}, z]$  a fraction  $\frac{a}{a+b}$  is misclassified as poor, while in fact they receive sufficient income (overestimation). Of all people in the range of expected income  $[z, \frac{z}{\lambda}]$  a fraction  $\frac{c}{c+d}$  is misclassified as non-poor, while in fact their income falls short of  $z$  (underestimation). In this specification of income risk

the underestimation effect is stronger than the overestimation effect, in the sense that the fraction of the population with an expected income just above the poverty line, which is misclassified as non-poor, is larger than the fraction of the population with expected income just below the poverty line, which is misclassified as poor. Furthermore, if  $z$  is to the left of the mode of the distribution, which holds for countries with moderate poverty, we find a larger share of the population just above the poverty line than just below the poverty line. Hence, for this case, we expect to find an overall underestimation of the head count ratio of poverty,  $\hat{H} - H > 0$ .

Table 4.1

## Risk adjusted headcount ratio of the poor

Country	Survey year	GNP <sup>a)</sup> per capita	H <sup>b)</sup>	$\hat{H}$			% error		
				$\lambda=0.75$	$\lambda=0.5$	$\lambda=0.25$	$\lambda=0.75$	$\lambda=0.5$	$\lambda=0.25$
Bolivia	1999	2193	14,4	14,9	16,5	19,9	3,2	12,8	27,6
Brazil	1998	6160	11,6	11,9	12,8	14,9	2,3	9,5	22,0
China	1999	3220	18,8	19,2	20,4	23,0	2,0	7,9	18,1
Colombia	1998	7500	19,7	20,0	20,9	22,7	1,4	5,6	13,2
Costa Rica	1998	6620	12,6	12,9	13,8	15,8	2,1	8,6	20,2
El Salvador	1998	2850	21,0	21,4	22,7	25,2	1,8	7,3	16,7
Ghana	1999	1793	44,8	45,1	46,0	47,8	0,6	2,7	6,2
Guatemala	1998	4070	10,0	10,3	11,5	13,9	3,2	12,7	28,3
Honduras	1998	2140	24,3	24,7	26,1	28,9	1,7	7,0	15,8
Indonesia	1999	2439	12,9	13,4	14,9	18,1	3,4	13,4	28,9
Madagascar	1999	766	49,1	49,5	50,8	52,8	0,9	3,3	7,0
Mexico	1998	7719	15,9	16,2	17,0	18,9	1,6	6,7	15,9
Panama	1998	6940	14,0	14,3	15,2	17,1	1,9	7,8	18,3
Paraguay	1998	3650	19,5	19,8	21,0	23,4	1,8	7,1	16,5
Russian Fed	1998	3950	7,1	7,4	8,5	11,0	4,2	16,6	35,7
Ukraine	1999	3142	2,9	3,2	4,2	7,0	8,8	31,5	58,8
Venezuela, RB	1998	8190	23,0	23,3	24,1	25,8	1,1	4,5	10,9
Yemen, Rep	1998	740	15,7	17,7	23,4	30,7	11,1	32,9	48,8
Zambia	1998	860	63,7	63,9	64,5	65,5	0,3	1,2	2,8

a) 1993 PPP dollars, *Sources*: for 1999: The World Bank (2001, 274-275); for 1998: The World Bank (2000, 230-231); for the deflator: The World Bank (2003, 188).

b) The international poverty line is set at  $z = 1.08$  in terms of 1993 PPP dollars. *Source*: The World Bank (2002, 236-237).

In order to examine the magnitude of underestimation we calculate risk adjusted headcount ratios for a sample of developing countries for which recent income surveys have been conducted (in 1998 and 1999) and reported in the "World Development Report". We consider three levels of risk where the worst possible case is that a person receives a fraction  $\lambda=0.75$ ,  $\lambda=0.5$  or  $\lambda=0.25$ , respectively, of her expected income.

Results are presented in table 4.1. Even for small risks ( $\lambda=0.75$ ) we find that the estimation error may exceed 10% and the ranking of countries may change. Yemen and Mexico swap ranks for  $\lambda=0.75$ . For large risks ( $\lambda=0.25$ ) underestimation effects can be substantial and are close to 60% in the case of Ukraine. Moreover, there are a number of rank reversals as countries with low average income and low inequality (e.g. Yemen) show larger errors than poor countries with higher inequality. For the latter (e.g. Zambia) the type I error almost compensates for the type II error. Note that these rank reversals occur even though we impose the same assumptions regarding the structure of risk on all countries. We use this ad hoc assumption to illustrate the potential importance of risk-adjusted poverty measurement. We would expect to find even stronger effects of income risks on the poverty ranking in international comparisons if cross country differences in individual exposure to risk were taken into account.<sup>5</sup> Underestimation errors for the Foster, Greer and Thorbecke index (not reported in the table) are found to be in the same order of magnitude. Rank reversals are somewhat less frequent.

## 5 Conclusions

Not only the poor but also all those who are at risk of becoming poor are affected by poverty. This should be reflected in a poverty measure. This paper develops refinements of existing poverty measures to account for income risk. The basic finding is that effects of income risks on the headcount ratio and the poverty gap ratio are ambiguous. The effects depend on the distribution of expected income across population and on the structure of income risk. However, the headcount ratio for societies with moderate poverty and a lognormal income distribution is likely to be underestimated if income risk is neglected. For the Foster-Greer-Thorbecke measure we find an unambiguous underestimation of expected poverty of standard measurement compared to a risk-adjusted measure.

Furthermore, we have calculated the effect of income risk on expected poverty for a sample of developing countries. Because of lack of empirical data we have used ad hoc assumptions for the distribution of income risk. The challenge for future research is to empirically estimate income risk distributions using, for example, panel data to identify *random* income change or questionnaires where individuals report subjective probability distributions of their income.

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<sup>5</sup> Gabbert et al. (2003) follow such an approach exploring international comparisons of food insecurity.

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