# Prospect Theory Applications in Finance

Nicholas Barberis
Yale University

April 2007

# Overview

- behavioral finance studies models in which some agents are less than fully rational
- "rationality" is typically taken to mean two things:
  - rational *beliefs*: update beliefs using Bayes' rule
  - rational *preferences*: make decisions according to EU, with a utility function defined over wealth or consumption

# Overview, ctd.

- one source of inspiration on plausible departures from rationality is the psychology literature
- psychology of beliefs
  - deviations from Bayes' rule
  - e.g. overconfidence, representativeness
- psychology of preferences
  - deviations from EU, concern for non-consumption utility
  - e.g. prospect theory, narrow framing, ambiguity aversion
- today, look at implications of *prospect theory* and *narrow framing* for asset prices and portfolio choice

## Prospect Theory

- we now have a lot of experimental evidence on attitudes to risk (gambles whose outcomes have known probabilities)
  - evidence reveals that people routinely violate EU
- there are many non-EU models that try to capture the experimental evidence
  - prospect theory (Kahneman and Tversky, 1979) is the best-known
  - it does the best job capturing the evidence
  - it is the only one that is explicitly descriptive

## Prospect Theory, ctd.

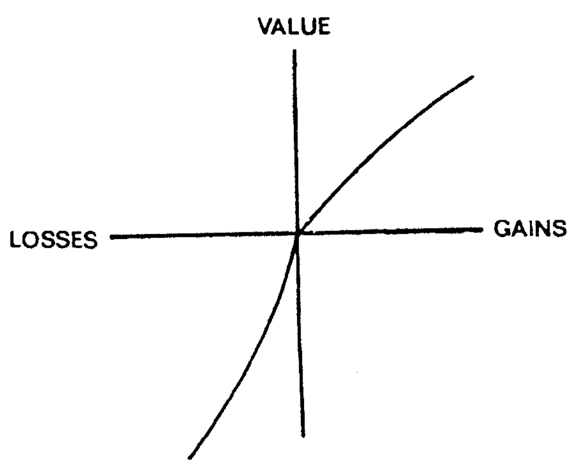
Consider the gamble $(x, p; y, q)$

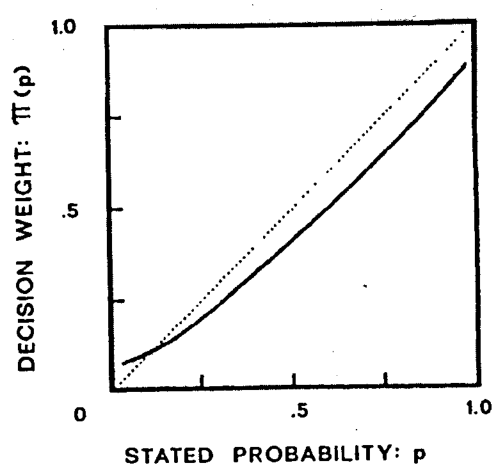
- under EU, it is assigned the value

$$pU(W + x) + qU(W + y)$$

- under Prospect Theory, it is assigned the value

$$\pi(p)v(x) + \pi(q)v(y)$$



## Prospect Theory, ctd.

*Four* key features:

- the carriers of value are *gains* and *losses*, not final wealth levels
  - compare $v(x)$ vs. $U(W + x)$
  - inferred from experimental evidence
  - also consistent with the way we perceive other attributes
- $v(\cdot)$ has a kink at the origin
  - captures a greater sensitivity to losses (even small losses) than to gains of the same magnitude
  - "loss aversion"
  - inferred from aversion to $(110, \frac{1}{2}; -100, \frac{1}{2})$
- $v(\cdot)$ is concave over gains, convex over losses
  - inferred from $(500, 1) \succ (1000, \frac{1}{2})$ and $(-500, 1) \prec (-1000, \frac{1}{2})$

## Prospect Theory, ctd.

- transform probabilities with a weighting function $\pi(\cdot)$ that:
  - overweights *low* probabilities
    * inferred from our simultaneous liking of lotteries and insurance, e.g. $(5,1) \prec (5000, 0.001)$ and $(-5,1) \succ (-5000, 0.001)$
  - is more sensitive to changes in probability at higher probability levels
    * e.g. $(3000,1) \succ (4000,.8)$ but $(3000,.25) \prec (4000,.2)$

Note:

- transformed probabilities should not be thought of as beliefs, but as decision weights
- they are a modeling device for capturing the experimental data
  - e.g. the preference for lottery-like gambles

# Cumulative Prospect Theory

- proposed by Tversky and Kahneman (1992)
- applies the probability weighting function to the *cumulative* distribution function:

$$(x_{-m}, p_{-m}; \ldots ; x_{-1}, p_{-1}; x_0, p_0; x_1, p_1; \ldots ; x_n, p_n),$$

where $x_i < x_j$ for $i < j$ and $x_0 = 0$, is assigned the value

$$\sum_{i=-m}^{n} \pi_i v(x_i)$$

$$\pi_i = \begin{cases} \pi(p_i + \ldots + p_n) - \pi(p_{i+1} + \ldots + p_n) & \text{for} \quad 0 \le i \le n \\ \pi(p_{-m} + \ldots + p_i) - \pi(p_{-m} + \ldots + p_{i-1}) & \quad\quad -m \le i < \end{cases}$$

- the agent now overweights the *tails* of a probability distribution
  - this preserves a preference for lottery-like gambles

# Cumulative Prospect Theory, ctd.

- Tversky and Kahneman (1992) also suggest functional forms for $v(\cdot)$ and $\pi(\cdot)$ and calibrate them to experimental evidence:

$$v(x) = \begin{cases} x^\alpha & \\ -\lambda(-x)^\alpha & \end{cases} \text{for} \begin{array}{c} x \geq 0 \\ x < 0 \end{array}$$

$$\pi(P) = \frac{P^\delta}{(P^\delta + (1-P)^\delta)^{1/\delta}}$$

with

$$\alpha = 0.88, \lambda = 2.25, \delta = 0.65$$

## Narrow framing

- in traditional models, an agent evaluates a new gamble by merging it with her pre-existing risks and checking if the combination is attractive
- narrow framing occurs when the new gamble is evaluated, to some extent, *in isolation*
  - get utility *directly* from the outcome of the gamble, not just indirectly from its contribution to total wealth
- early example of narrow framing appears in Tversky and Kahneman (1981)
  - term is first used in Kahneman and Lovallo (1993)
- very similar to "mental accounting"

# Narrow framing, ctd.

- Barberis, Huang, and Thaler (2006) argue that the rejection of $(110, \frac{1}{2}; -100, \frac{1}{2})$ is not only evidence of loss aversion, but of narrow framing as well
- if the agent has pre-existing risk, it's difficult to explain the rejection of the gamble without appealing to narrow framing
  - EU models and wide range of non-EU models have a hard time doing so, including even non-EU models with kinks
  - for someone who frames "broadly", above gamble is attractive, even when utility function is kinked

# Interpreting narrow framing

- how should narrow framing be interpreted?
- two possibilities:
  - it is related to non-consumption utility, e.g. regret, which is plausibly associated with a narrow frame
  - it stems from an *intuitive* attempt to maximize consumption utility
    - intuition uses “accessible” information, and the most accessible information may be about narrow components of wealth (Kahneman, 2003)

# Applications of prospect theory and narrow framing

*Probability weighting function*

I. Pricing of skewness

*Concavity/convexity of value function over gains/losses*

II. Disposition effect

*Loss aversion*

III. Equity premium

IV. Stock market non-participation

Theme:

- it can be surprisingly hard to get new implications out of prospect theory
  - to generate interesting predictions, often need additional machinery, e.g. narrow framing

## Models and References

*Probability weighting function*

- static model where preferences consist only of a prospect theory term

Barberis and Huang (2007a, WP), "Stocks as Lotteries: The Implications of Probability Weighting for Security Prices"

*Concavity/convexity of value function over gains/losses*

- dynamic model where preferences consist only of a prospect theory term

Barberis and Xiong (2006a, WP), "What Drives the Disposition Effect? An Analysis of a Long-Standing Preference-Based Explanation"

# Models and References, ctd.

*Loss aversion*

- dynamic model where preferences also include a utility of consumption term

Barberis and Huang (2007b, HEP), "The Loss Aversion / Narrow Framing Approach to the Equity Premium Puzzle"

Barberis, Huang, and Santos (2001a, QJE), "Prospect Theory and Asset Prices"

Barberis and Huang (2001b, JF), "Mental Accounting, Loss Aversion, and Individual Stock Returns"

Barberis, Huang, and Thaler (2006b, AER), "Individual Preferences, Monetary Gambles, and Stock Market Participation"

Barberis and Huang (2004, WP), "Preferences with Frames: A New Utility Specification that Allows for the Framing of Risks"

## Models and References, ctd.

In all cases:

- have to decide on a frame, narrow or broad
  - which asset do the gains and losses refer to?
- then decide on the precise definition of the gain/loss
  - e.g. what is the reference point and how does it move?

# I. Pricing of skewness (PW)

Barberis and Huang (2007a)

- single period model; a risk-free asset and $J$ Normally distributed risky assets
- agents have identical expectations about security payoffs
- agents have identical CPT preferences
  - defined over gains/losses in *wealth* (i.e. no narrow framing)
  - reference point is initial wealth scaled by riskless rate, so utility defined over $\hat{W} = \tilde{W}_1 - W_0 R_f$
  - full specification is:

    $$V(\hat{W}) = \int_{-\infty}^{0} v(W)\, d\pi(P(W)) - \int_{0}^{\infty} v(W)\, d\pi(1 - P(W))$$

    (continuous distribution version of Tversky and Kahneman, 1992)

# I. Pricing of skewness (PW), ctd.

Then:

- the CAPM holds
  - FOSD holds $\Rightarrow$ all investors are on the MVE frontier
- but if we introduce a small, independent, positively skewed security, it earns a *negative* excess return
  - skewness itself is priced, in contrast to concave EU model where only coskewness with market matters
- equilibrium involves *heterogeneous holdings* (assume short-sale constraints)
  - some investors hold a large, undiversified position in the new security
  - others hold no position in it at all
  - heterogeneous holdings arise from non-unique global optima, not from heterogeneous preferences
- since the new security contributes skewness to the portfolios of some investors, it is valuable, and so earns a low average return

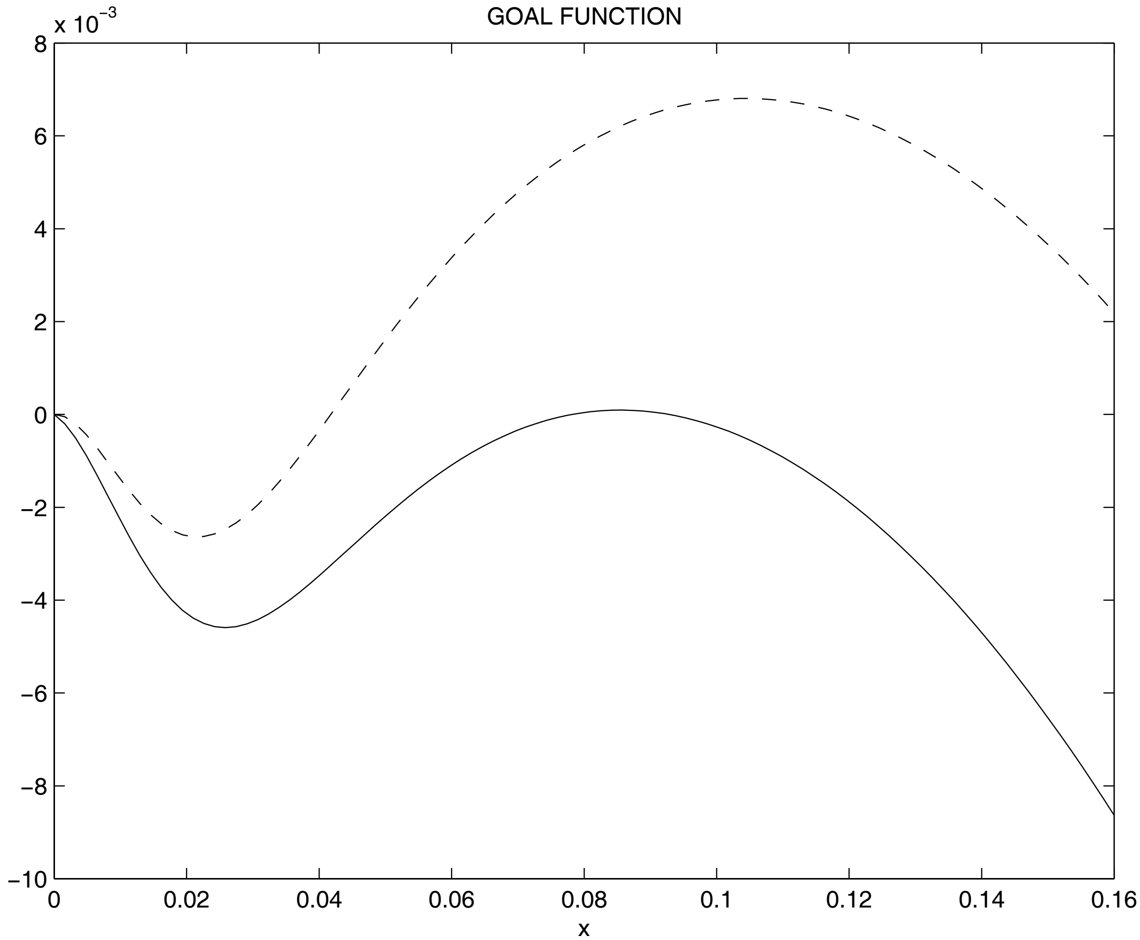



Figure 3. The figure shows the utility that an investor with cumulative prospect theory preferences derives from adding a position $x$ in a positively-skewed security to his current holdings of a normally distributed market portfolio. The dashed line corresponds to a higher mean return on the skewed security.

# I. Pricing of skewness (PW), ctd.

- this only works if the new security is highly skewed
  - otherwise, would need too undiversified a position in order to add skewness to portfolio
- results hold:
  - even if there are *many* skewed securities
  - even if short sales are allowed
  - even if arbitrageurs are present

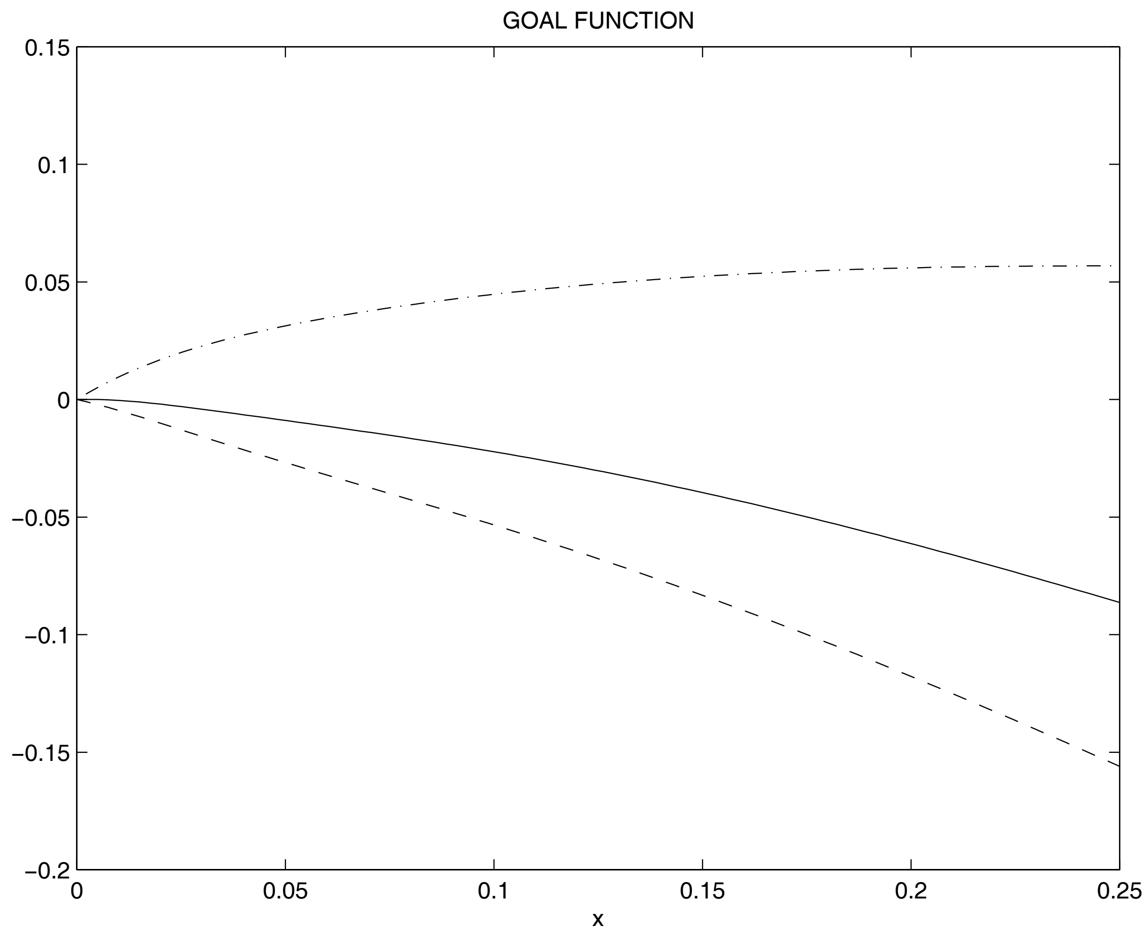



Figure 4. The figure shows the utility that an investor with cumulative prospect theory preferences derives from adding a position $x$ in a positively-skewed security to his current holdings of a normally distributed market portfolio. The three lines correspond to different mean returns on the skewed security.

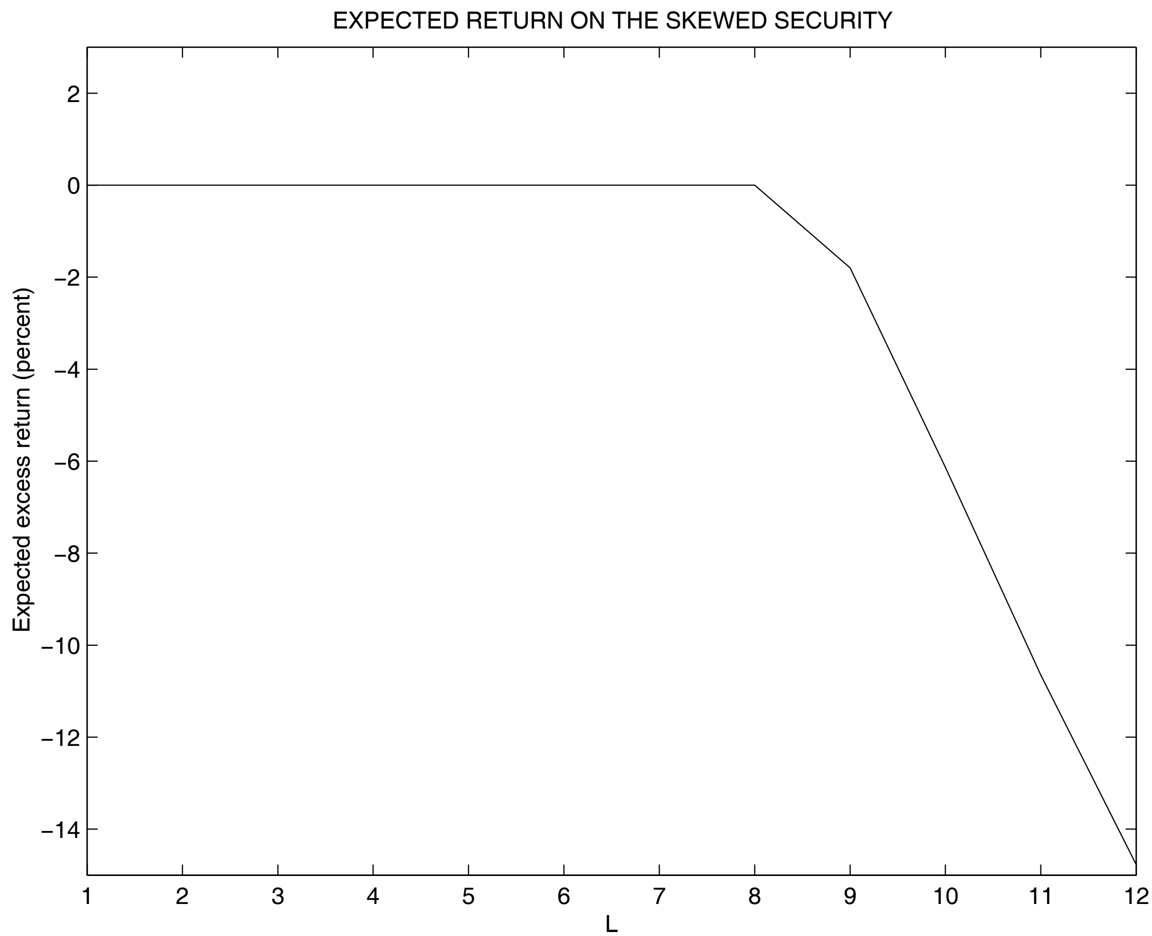



Figure 5. The figure shows the expected return in excess of the risk-free rate earned by a small, independent, positively skewed security in an economy populated by investors who judge gambles according to cumulative prospect theory, plotted against a parameter of the the security's return distribution. The security earns a gross return of 0 with high probability and of L with low probability.

# I. Pricing of skewness (PW), ctd.

*Applications*

- low average returns on IPOs
  - IPO returns are highly positively skewed
- diversification discount
  - Mitton and Vorkink (2007)
- under-diversification
  - Mitton and Vorkink (2006) find that undiversified individuals hold stocks that are more positively skewed than the average stock
- other:
  - low average return to "private equity"
  - low average return on distressed stocks
  - pricing of out-of-the-money options

## II. Disposition effect (CC)

- Odean (1998) studies the trading activity, from 1987-1993, of 10,000 households with accounts at a large discount brokerage firm
- whenever an investor sells shares of a stock, classify each of the stocks in her portfolio on that day as one of:
  - "realized gain", "realized loss", "paper gain", or "paper loss"
- add up total number of realized gains and losses and paper gains and losses over all accounts over the sample, and compute:

$$\text{PGR} = \frac{\text{no. of realized gains}}{\text{no. of realized gains} + \text{no. of paper gains}}$$

$$\text{PLR} = \frac{\text{no. of realized losses}}{\text{no. of realized losses} + \text{no. of paper losses}}$$

(e.g. PGR is "proportion of gains realized")

- the disposition effect is the finding that PGR $>$ PLR
  - specifically, $0.148 = \text{PGR} > \text{PLR} = 0.098$

## II. Disposition effect (CC), ctd.

The most obvious potential explanations fail to capture important features of the data

- e.g. informed trading
  - the subsequent return of winners that people sell is *higher* than that of losers they hold on to
- e.g. taxes, rebalancing, transaction costs

Two non-standard hypotheses have gained prominence

- an irrational belief in mean-reversion
- an explanation based on prospect theory and narrow framing

At first glance, prospect theory and narrow framing do seem to generate a disposition effect

- in a formal model, however, Barberis and Xiong (2006a) find that prospect theory can also predict the *opposite* of the disposition effect

# II. Disposition effect (CC), ctd.

- consider a simple portfolio choice setting
  - $T + 1$ dates: $t = 0, 1, \ldots, T$
  - a risk-free asset, gross return $R_f$ each period
  - a risky asset with an i.i.d binomial distribution across periods:

$$R_{t,t+1} = \begin{cases} R_u > R_f & \text{with probability } \frac{1}{2} \\ R_d < R_f & \text{with probability } \frac{1}{2} \end{cases}, \text{ i.i.d.}$$

- the investor has prospect theory preferences defined over her "gain/loss"
  - simplest definition of gain/loss is trading profit between 0 and $T$, i.e. $W_T - W_0$
  - we use $W_T - W_0 R_f^T$
  - call $W_0 R_f^T$ the "reference" wealth level

# II. Disposition effect (CC), ctd.

The investor therefore solves

$$\max_{x_0, x_1, \ldots, x_{T-1}} E[v(\Delta W_T)] = E[v(W_T - W_0 R_f^T)]$$

where

$$v(x) = \begin{cases} x^\alpha & \\ -\lambda(-x)^\alpha & \end{cases} \text{for} \begin{array}{c} x \geq 0 \\ x < 0 \end{array},$$

subject to

$$\begin{aligned} W_t &= (W_{t-1} - x_{t-1} P_{t-1}) R_f + x_{t-1} P_{t-1} R_{t-1,t} \\ W_T &\geq 0 \end{aligned}$$

- using the Cox-Huang (1989) methodology, can derive an analytical solution for any number of trading periods

# II. Disposition effect (CC), ctd.

*Example*

- set $(P_0, W_0) = (40, 40)$
- set $R_f = 1$ and $T = 4$
- set $(\alpha, \lambda) = (0.88, 2.25)$
- to set $(R_u, R_d)$, think of the interval from 0 to $T$ as a year, and choose sensible values for the stock's *annual* mean and standard deviation $(\mu, \sigma)$
  - back out the implied $(R_u, R_d)$
- e.g. $(\mu, \sigma) = (1.1, 0.3)$
  - implies $(R_u, R_d) = (1.16, 0.89)$

| $P_{t,i}$ | | | | | $q_{t,i}$ | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 72.9 | | | | | 0.46 |
| | | | 62.7 | | | | | 0.56 | |
| | | 54.0 | | 55.6 | | | 0.68 | | 0.66 |
| | 46.5 | | 47.9 | | | 0.83 | | 0.80 | |
| 40 | | 41.2 | | 42.4 | 1 | | 0.97 | | 0.94 |
| | 35.5 | | 36.5 | | | 1.18 | | 1.14 | |
| | | 31.4 | | 32.4 | | | 1.38 | | 1.34 |
| | | | 27.9 | | | | | 1.62 | |
| | | | | 24.7 | | | | | 1.91 |

| $x_{t,i}$ | | | | | $W_{t,i}$ | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | - | | | | | 163.39 |
| | | | 6.8 | | | | | 94.70 | |
| | | 3.5 | | - | | | 64.25 | | 46.47 |
| | 1.8 | | 0.5 | | | 50.75 | | 42.87 | |
| 1.7 | | 0.2 | | - | 40 | | 41.27 | | 40.34 |
| | 1.5 | | 0.0 | | | 32.45 | | 40.15 | |
| | | 2.7 | | - | | | 26.26 | | 40.02 |
| | | | 5.2 | | | | | 16.51 | |
| | | | | - | | | | | 0 |

# II. Disposition effect (CC), ctd.

Does prospect theory predict a disposition effect?

- construct a simulated dataset of how 10,000 prospect theory investors trade $N_S$ stocks over $T$ periods
  - simulate a $T$-period path through the binomial tree for $10,000 \times N_S$ stocks
  - for each path, earlier analysis tells us how the investor trades along the path
- now follow Odean's (1998) exact methodology for computing PGR and PLR
  - if PGR $>$ PLR, there is a disposition effect
- parameter values:
  - set $(P_0, W_0) = (40, 40)$ for each stock
  - set $R_f = 1$ and $\sigma = 0.3$
  - set $(\alpha, \lambda) = (0.88, 2.25)$
  - Barber and Odean (2000) report $N_S = 4$
  - range of values of $\mu$ and $T$

| $\mu$ | $T=2$ | $T=4$ | $T=6$ | $T=12$ |
|---|---|---|---|---|
| 1.03 | - | - | - | .55/.50 |
| 1.04 | - | - | .54/.52 | .54/.52 |
| 1.05 | - | - | .54/.52 | .59/.45 |
| 1.06 | - | .70/.25 | .54/.52 | .58/.47 |
| 1.07 | - | .70/.25 | .54/.52 | .57/.49 |
| 1.08 | - | .70/.25 | **.48/.58** | **.47/.60** |
| 1.09 | - | **.43/.70** | **.48/.58** | **.46/.61** |
| 1.10 | **0.0/1.0** | **.43/.70** | **.48/.58** | **.36/.69** |
| 1.11 | **0.0/1.0** | **.43/.70** | **.49/.58** | **.37/.68** |
| 1.12 | **0.0/1.0** | **.28/.77** | **.23/.81** | **.40/.66** |
| 1.13 | **0.0/1.0** | **.28/.77** | **.24/.83** | **.25/.78** |

## II. Disposition effect (CC), ctd.

Why do we always see the *opposite* of the disposition effect in the 2-period case?

- think about the investor's strategy at time 1
  - focus on situations in which the expected risky asset return is not too low
- after a gain at time 1, the investor takes a position such that, after a poor time 2 return, she ends up with a small gain
  - since $v(\cdot)$ is only *mildly* concave over gains, she gambles to the edge of the concave region, but no further
- after a loss at time 1, the investor takes a position such that, after a good time 2 return, she again ends up with a small gain
  - since $v(\cdot)$ is convex over gains, she gambles to the edge of the convex region, but not much beyond

# II. Disposition effect (CC), ctd.

So why does the disposition effect fail?

- for the investor to buy the stock at all at time 0, in spite of her loss aversion, it must have a relatively high expected return
  - this implies that the time 1 gain is larger than the time 1 loss in magnitude
  - it also implies that, after a gain, the investor gambles to the edge of the concave region
- but it takes a *larger* position to gamble to the edge of the concave region after a gain, than it does to gamble to the edge of the convex region, after a loss

  ⇒ the investor takes more risk after a gain than after a loss, contrary to the disposition effect

## II. Disposition effect (CC), ctd.

Why is the disposition effect more likely to hold for high $T$ or low $\mu$?

- for high $T$, the kink is smoothed out and the investor might buy at time 0 even if the expected risky asset return is very low
- in this case, after a gain, she will take a *small* position in the risky asset
  - after a loss, she will still gamble to the edge of the convex region

  $\Rightarrow$ the disposition effect may hold

This suggests some testable predictions:

- the disposition effect is more likely to hold among stocks with characteristics associated with lower average returns
- traders who buy with a higher $T$ in mind are more likely to exhibit a disposition effect

# III. Equity premium (LA)

- Benartzi and Thaler (1995) argue that loss aversion and narrow framing may help to address the equity premium puzzle
  - specifically, loss aversion over *annual changes in the value of stock market holdings*
- to address the equity premium properly, need to introduce consumption in a non-trivial way
  - preferences must include "utility of consumption" term alongside the prospect theory term
- two ways of doing this:
  - Barberis, Huang, and Santos (2001a)
  - Barberis and Huang (2004)
- Barberis and Huang (2007b) reviews both methods

# III. Equity premium (LA), ctd.

*Method I* (Barberis, Huang, and Santos, 2001a)

- intertemporal model; three assets: risk-free ($R_{f,t}$), stock market ($R_{S,t+1}$), non-financial asset ($R_{N,t+1}$)
- representative agent maximizes:

$$E_0 \sum_{t=0}^{\infty} \left[ \rho^t \frac{C_t^{1-\gamma}}{1-\gamma} + b_0 \rho^{t+1} \overline{C_t}^{-\gamma} v(G_{S,t+1}) \right]$$

$$\begin{aligned} G_{S,t+1} &= \theta_{S,t}(W_t - C_t)(R_{S,t+1} - 1) \\ v(x) &= \begin{cases} x & \text{for } x \geq 0 \\ \lambda x & \phantom{\text{for }} x < 0 \end{cases}, \lambda > 1 \end{aligned}$$

  - frame stock market narrowly
  - reference point is updated annually
  - $v(\cdot)$ captures only loss aversion
  - two interpretations: consumption vs. non-consumption utility, rational vs. intuitive thinking
- for "reasonable" parameters, get substantial equity premium

## III. Equity premium (LA), ctd.

- loss aversion over annual changes in value of stock holdings ⇒ high equity premium
  - original idea in Benartzi and Thaler (1995)
- annual evaluation period is important
- although BT don't emphasize this, narrow framing of stocks is also very important
  - loss aversion over annual changes in *total* wealth doesn't give as large a premium

# III. Equity premium (LA), ctd.

*Method II* (Barberis and Huang, 2004)

- start from standard recursive specification

$$\begin{aligned} V_t &= W(C_t, \mu(V_{t+1})) \\ W(C, x) &= ((1-\beta)C^\rho + \beta x^\rho)^{\frac{1}{\rho}},\ 0 < \beta < 1,\ 0 \neq \rho < 1 \\ \mu(x) &= (E(x^\zeta))^{\frac{1}{\zeta}} \end{aligned}$$

- can adjust this to incorporate narrow framing

$$V_t = W\left(C_t, \mu(V_{t+1}) + b_{i,0} \sum_i E_t(v(G_{i,t+1}))\right)$$

- in 3-asset context from before:

$$\begin{aligned} V_t &= W\left(C_t, \mu(V_{t+1}) + b_0 E_t(v(G_{S,t+1}))\right) \\ G_{S,t+1} &= \theta_{S,t}(W_t - C_t)(R_{S,t+1} - 1) \\ v(x) &= \begin{cases} x & \text{for } x \geq 0 \\ \lambda x & \phantom{\text{for }} x < 0 \end{cases},\ \lambda > 1 \\ \zeta &= \rho \end{aligned}$$

## III. Equity premium (LA), ctd.

- this specification is better than Method I
  - does not require aggregate consumption scaling $\bar{C}$
  - is tractable in partial equilibrium
  - admits an explicit value function $\Rightarrow$ easy to check attitudes to monetary gambles
- can now show that for parameter values that predict reasonable attitudes to large and small-scale monetary gambles, get substantial equity premium
- Barberis, Huang, and Santos (2001a) also build in dynamic aspects of loss aversion
  - "house money effect"
  - generates high volatility, predictability, in addition to equity premium

## IV. Stock market non-participation (LA)

- in general, combination of loss aversion and narrow framing predicts aversion to an independent, actuarially favorable gamble with roughly equiprobable gains and losses
  - by looking at gamble in isolation, neglect diversification benefits
- potential applications:
  - stock market non-participation (Barberis, Huang, and Thaler, 2006b)
  - low number of stocks held directly
  - home bias

# IV. Under-diversification (LA), ctd.

- narrow framing is crucial here
  - loss aversion over *total wealth* does not predict stock market non-participation (Barberis, Huang, and Thaler, 2006b)
  - even a loss averse agent enjoys the diversification benefits that a position in equities add to her other risks
- Dimmock (2005) tests the loss aversion / narrow framing view of stock market participation

## Summary

- probability weighting $\Rightarrow$ pricing of skewness
  - no narrow framing needed
- concavity/convexity of value function $\Rightarrow$ disposition effect (sometimes!)
  - need narrow framing
- loss aversion $\Rightarrow$ equity premium, under-diversification
  - need narrow framing

(Recall the theme mentioned earlier!)

Future work?

- test prospect theory hypotheses for various facts
- build up theoretical foundations of behavioral finance